BIOLOGY AND THE ARGUMENTS OF UTILITY*

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Abstract

Why did evolution not give us a utility function that is offspring alone? Why do we care intrinsically about other outcomes, food, for example, and what determines the intensity of such preferences? A common view is that such other outcomes enhance fitness and the intensity of our preference for a given outcome is proportional to its contribution to fitness. We argue that this view is inaccurate. Specifically, we show that in the presence of informational imperfections, the evolved preference for a given outcome is determined by the individual's degree of ignorance regarding its significance. Our model sheds light on imitation and prepared learning, whereby some peer attitudes are more influential than others. Testable implications of the model include systematically biased choices in modern times. Most notably, we apply the model to help explain the demographic transition.

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1 Introduction

Despite a steady trickle of research on the issue over the last 20 years, it remains unconventional to consider the biological basis of utility or preferences.[1] This approach holds out the promise of generating a utility function with the key economic properties of being exogenous and invariant. At first blush, such a biologically derived utility would value commodities in accordance with their impact on fitness—we should value food, for example, in terms of its marginal contribution to biological success. However, on reflection, a serious conceptual problem arises—why have we been made to care about *anything other* than offspring? Why, that is, if we are rational and intelligent, are we not programmed to like *only* offspring and then to treat food, for example, as purely instrumental, as a means to an end? In modern times, indeed, we readily sacrifice expected offspring to increase consumption of other commodities. The recent "demographic transition," during which incomes rose but fertility fell is dramatic *prima facie* evidence on this score.

We consider a solution to this conundrum in terms of information. Our theory has two key components:

First, the individual has two signals available concerning the state of the world. On the one hand, there are relevant aspects of the environment that are "recurrent signals" in the sense of having a long evolutionary history. For example, sunlight has long had an effect in aiding Vitamin D production and thereby enhancing health and fitness.[2] Nature then has had the opportunity to incorporate a liking for the sun in the utility function. On the other hand, there are relevant aspects of the environment that are "transient signals" in the sense of being local in time and space, and having arisen only rarely before, or perhaps never. These signals might concern likely locations in which to sunbathe, for example. Natural selection could not have incorporated these aspects of the environment in the utility function.

Secondly, the individual has arbitrary beliefs concerning the implications of the two signals, but is otherwise rational. In particular, the individual maximizes the expectation of his utility, conditioning on both the recurrent and transient signals, using these arbitrary beliefs.

[1] Becker (1976) was an early exponent of the need to link economics to biology. More particularly, Robson (2001b) argues that a utility function serves as a method for Nature to partially decentralize control, thus achieving a flexible response to novelty. There is a distinct literature that considers how preferences that are not fitness might evolve to yield a fitness advantage in particular games. See, for example, Güth &Yaari (1992) and Dekel, Ely & Yilankaya (2007).

[2] The full importance of Vitamin D was only recently recognized.

We derive two general results. First, despite Nature's inability to condition the individual's utility on the transient signal, and despite the individual's holding arbitrary beliefs, Nature can shape the individual's utility such that the equilibrium action chosen by the individual is the best possible in the sense of maximizing expected fitness conditional on both signals. Secondly, the resulting marginal utility of an action is determined not by the marginal contribution of that action to fitness, but by the individual's degree of ignorance regarding such contribution.

Our model sheds new light on the demographic transition. Specifically, we argue that humans have evolved direct concerns for offspring quality and for imitation of peer fertility that are evolutionarily suboptimal in modern circumstances. We at once buttress the approach of Cavalli-Sforza and Feldman (1981) and that of Becker and Lewis (1973).

Our theoretical approach is an instance of the principal-agent problem. In this interpretation, the principal (Nature), maximizes the productivity of the agent (individual) by choosing the agent's utility function. Papers that also employ the principal-agent metaphor, explicitly or implicitly, include Binmore (1994), Frederick & Lowenstein (1999), Robson (2001a), Samuelson (2004), Rayo & Becker (2007), Netzer (2009), and Herold & Netzer (2010). While the principal-agent perspective is illuminating, there are no formal techniques or results that can be directly imported from the existing literature.[3] In fact, the specific principal-agent problem we consider is not meaningful in conventional economic applications. Most significantly, we assume that (i) the principal has the power to fully shape the agent's preferences, (ii) all actions of the agent are contractible in the sense that utility depends directly on the action, and (iii) the principal has information that cannot be directly communicated to the agent, despite the parties having parallel interests in the information.

A paper that can be described in analogous terms is Samuelson and Swinkels (2006), who also consider an environment in which both Nature and the individual possess relevant information. In an ideal case, the individual would maximize expected fitness. In their work, unlike ours, there is an emphasis on second-best solutions that provide a rationale for behavioral biases. We discuss the key issue of why the Samuelson and Swinkels approach leads to a second-best situation but ours does not in Section 2.1.

Our focus throughout is on "primary" rather than "secondary" arguments of utility. That is, we consider arguments that are desired as ends in themselves rather than as means to

[3] In spirit, ours is a model of delegation. See, for example, Holmstrom (1984), Aghion & Tirole (1997), Melumad, Mookherjee & Reichelstein (1997), Dessein (2002), Alonso & Matouschek (2008), and Armstrong & Vickers (2010).

an end. There are many primary arguments, of course. For example, Linden's list includes food, prestige, temperature, a view, and sex (see Linden, 2011). Bentham lists 26 categories of "simple" pleasures and pains (Bentham, 1789). Perhaps the most salient example of a secondary argument is money, which is fundamentally only a means to an end from the perspective of the individual.[4]

The remainder of the paper is organized as follows:

Section 2 presents the basic model. We dismiss as unrealistic the possibility that the individual's beliefs match the true distribution. This leaves us with a mechanism in which Nature shapes utility in the light of her recurrent information and the individual maximizes expected utility conditional on arbitrary beliefs. Section 2.1 considers the problem of existence of a utility function that guarantees first-best actions. Although existence may be a delicate issue in a general continuous formulation, it is a straightforward question in a discrete approximation.

Section 3 reverts to a continuous formulation, while also imposing a monotone structure on the problem, for concreteness. The main result of the paper is Theorem 1 which proves that optimal choice can be supported by a utility function that has a simple additive form. Section 3 intertwines the theoretical argument with two simple but economically relevant examples. The first of these involves choice of labor-leisure; the second involves the quantity-quality tradeoff for offspring.

Section 4 considers how the evolutionary re-shaping of utility would result in systematically inefficient choices in a modern environment, assuming that this environment now entails accurate beliefs. We propose this observation as the key testable empirical implication of the present model. We use the quantity-quality example to illustrate how inefficient choices would arise, with an application to the demographic transition that occurred during the industrial revolution.

Section 5 shows how the present framework can be readily adapted to consider the evolution of interdependent preferences. We show how imitation, a desire for conformity, would arise. We then focus on a model that is generalized in that it allows an infinite sequence of individuals, each observing the action of a predecessor, but simplified in that it assumes quadratic fitness and normal signals. This model yields insights into the intriguing psycho-

[4] Not all of Bentham's categories seem clearly primary. For example, he nominates wealth as a simple pleasure, but then defends this choice in terms of what the money can buy.

Since money is a very familiar means, it induces a rather automatic response. It was once thought that the fMRI signature of money could not be distinguished from that of sex. However, Sescousse *et al.* (2010) show that money and sex have subtly distinct fMRI signatures, reflecting the instrumental role of money.

logical phenomemon of “prepared learning.” In modern circumstances, with accurate beliefs, there is overimitation, providing support for the analysis by Cavalli-Sforza and Feldman (1981) of the demographic transition. Section 5.1 provides an account of the demographic transition that integrates overimitation with an exaggerated concern with offspring quality. Finally, Section 6 concludes.

2 Model

There are two players: a *principal* (Nature) and an *agent* (the individual). The agent faces a one-shot opportunity to produce fitness $y \in \mathbb{R}$ (representing quality-adjusted offspring). Fitness is a function of the individual’s choice of a vector of actions $x \in \mathbb{R}^N$ and an underlying state $\sigma \in \mathbb{R}$:

$$y = \varphi(x, \sigma).$$

The players have only partial knowledge of the state. That knowledge has two components: $s \in \mathbb{R}$ and $t \in \mathbb{R}$. These components represent, respectively, recurrent and transient aspects of the environment. The essential distinction is that the utility function given to the individual can depend on the recurrent component, s, but not on the transient component, t. From Nature’s point of view, the (“true”) distribution of σ conditional on (s,t) is given by the pdf $f(\sigma \mid s,t)$; from the individual’s point of view, the (“subjective”) distribution of σ conditional on t is given by the pdf $g(\sigma \mid s,t)$. We allow Nature to shape utility in the light of these pdfs f and g.

A more detailed interpretation of s and t is as follows. On the one hand, s represents observed aspects of the environment whose implications have remained constant over evolutionary time. For example, s might represent exposure to sunlight. Exposure to UV light, in particular, aids in the production of Vitamin D, which is essential for health. The pdf f incorporates the true effect of sunlight on fitness. The individual, however, can be largely or completely ignorant of this causal chain, having arbitrary beliefs as captured by the pdf g. The recurrent nature of this causal chain means, however, that Nature can shape the utility function in the light of s.

The reason we explicitly include the signal s is to allow it to change over time. For example, if you are warm, cold drinks are helpful; if you are cold, hot. Evolution has made the appropriate drink seem pleasurable in each circumstance. Kandel *et al.* (2000) describe the neurological basis of this phenomenon. In terms of our model, this phenomenon corresponds to allowing utility to depend upon an s that can have a number of possible realizations that vary over time. The signal s is recurrent, since each of these circumstances is evolutionarily familiar. This interpretation of a recurrent signal is especially useful in the model of imitation in Section 5. There, the action of a predecessor has a number of possible values, but each of these is taken to be evolutionarily familiar, so evolved utility can be made a function of this action. Although we need the signal s for the sake of generality and to facilitate extensions of the model, the basic results can be appreciated by suppressing s.

On the other hand, t represents observed aspects of the environment that may be important but have a one-off quality. For instance, a hunter observes the exact position and strength of his prey, as well as the current abundance of alternative sources of food. By observing these precise local conditions, in the language of Binmore (1994), the individual serves as Nature's "eyes." We assume that the rarity, or even complete novelty, of such an aspect means that Nature cannot condition utility on t.

We have assumed that, although Nature cannot make utility dependent on t, Nature does shape utility in the light of f and g. In what sense is this a plausible assumption? If t represents completely unrestricted novelty, it is clearly not. For it to be plausible, there must be a suitable restriction on novelty. Specifically, f and g need to be sufficiently simple functions. Consider the following example. Fitness is given by $\sigma x - (1/2)x^2$, where x is effort by the individual and σ is the state of the world. The individual receives a signal t. (For simplicity, we abstract from the signal s.) The true distribution of the state given t is given by a pdf f with the property that $E_f(\sigma|t) = t$. It follows that the true optimal choice of x is $x^*(t) = t$. Suppose the individual's belief concerning the distribution of the state given t is given by a pdf g with the property that $E_g(\sigma|t) = Mt$ where $M > 1$. That is, the individual has overly optimistic beliefs about the marginal benefit of effort. If the individual maximized fitness in the light of the pdf g, he would expend an excessive level of effort $\hat{x}(t) = Mt > t$. However, the individual can be induced to choose $x^*(t)$ if the evolved utility is $\sigma x - (M/2)x^2$. Thus, a simple evolutionarily optimal response to the individual's misperception of the state is to have the individual perceive an inflated cost of effort. We analyze a more general version of this example, as Example 1, in Section 3 below. For the present purpose, the relevant feature is that, although utility here is independent of the realization of t, the utility construction works for all such realizations.

The model proceeds in three stages:

1. Nature selects a utility function $U(x, y, s)$ for the individual which can depend on the

choice, x, realized fitness, y, on the, as yet, undetermined realization of s, but not on that of t. The goal of the principal is to maximize the agent's true expected fitness, as expressed via the pdf f.

2. The signals s, t are realized.

3. The agent learns his utility function U, observes s, t and chooses x. The goal of the agent is to maximize his expected utility conditional on the information available to him, as expressed in the pdf g.

4. The state σ is drawn and the payoffs of both players – fitness for the principal and utility for the agent – are realized.

We interpret this setting as a metaphor for the long-run outcome of an evolutionary process in which the utility functions of humans are heritable and are the object of natural selection. Over time, through a trial-and-error process, those individuals endowed with utility functions that best promote their own fitness dominate the population. Rather than explicitly modelling such trial-and-error process, we suppose Nature can directly "choose" a fitness-maximizing utility function for each human being.

From the principal's perspective, the ideal choice of x solves

$$\max_{x} \quad \mathbb{E}_f \left[y \mid s, t \right], \tag{1}$$

where $\mathbb{E}_f$ means that the expectation is taken with respect to the true pdf f. For simplicity, we assume that, for each pair (s, t), this problem has a unique solution, denoted $x^*(s, t)$. If a function U implements $x^*(s, t)$ for all (s, t), we say it is optimal.

The individual is fully informed

If the individual is fully informed, so that $g = f$, his objective is

$$\max_{x} \quad \mathbb{E}_f \left[U(x, y, s) \mid s, t \right]. \tag{2}$$

A trivially optimal utility function is then

$$U(y) \equiv y,$$

which perfectly aligns the agent's objective (2) with the principal's objective (1).

Such resolution, however, is not a realistic description of humans. Most obviously, perhaps, we do not value only offspring intrinsically, viewing sex, for example, purely as an instrumental means to the end of producing more offspring. Less obviously, but perhaps more convincingly, consider how the experimental results of Wedekind *et al.* (1995) imply that our utility functions have arguments other than fitness. These results are that males with compatible immune systems appear to smell good to women. In the language of Barash (1979), a pleasant smell produces a "whispering within" that motivates them to select such mates. This amounts to this smell being an argument of utility. We are not born knowing that compatibility between parental immune systems is relevant to the fitness of offspring, or with any knowledge about how to check such compatibility.

The individual is not fully informed

When the individual holds arbitrary beliefs g his problem becomes

$$\max_x \quad \mathbb{E}_g \left[U(x, y, s) \mid s, t \right] \quad = \quad \max_x \quad \int U(x, y, s) g(\sigma \mid s, t) d\sigma,$$

where $\mathbb{E}_g$ means that the expectation is taken with respect to the subjective pdf g, and where $y = \varphi(x, \sigma)$. Note that t affects the individual's decision exclusively though the conditional distribution of σ, whereas s serves also as a parameter of the utility function.

In the remainder of the paper, we restrict attention to this scenario. We will show that this mechanism is constrained optimal, for hunter-gatherers, so there would have been no selection pressure for modification.

2.1 First-Best Implementation – Finite Case

In general, a basic theoretical question is whether there exists a function $\bar{U}$ that depends only on x, σ, and s and that satisfies the integral equation

$$\int \varphi(x, \sigma) f(\sigma | s, t) d\sigma = \int \bar{U}(x, \sigma, s) g(\sigma | s, t) d\sigma,$$

where the functions $\varphi(x, \sigma)$, $f(\sigma | s, t)$ and $g(\sigma | s, t)$ have been specified exogenously. If $y = \varphi(x, \sigma)$ were strictly monotonic in σ, for each x, then the existence of a function $U(x, y, s)$ that implements the fitness-maximizing action $x^*(s, t)$ for all s, t would be a consequence.

However, the existence of a solution for $\bar{U}(x, \sigma, s)$ to such a "Fredholm equation of the first kind" (Hochstadt, 1973) is a delicate issue.[5]

The choice of a continuous formulation over a discrete one here is mainly a matter of convenience. Indeed, from a conceptual point of view, a discrete formulation seems unobjectionable. In such a formulation, existence can be readily addressed. Suppose, then, that σ and t are restricted to $\{1, ..., S\}$. Given s, the problem is to find $\bar{U}(x, \sigma, s)$ such that

$$\sum_{\sigma} \overbrace{\varphi(x, \sigma)}^{1\times S} \overbrace{f(\sigma|s, t)}^{S\times S} = \sum_{\sigma} \overbrace{\bar{U}(x, \sigma, s)}^{1\times S} \overbrace{g(\sigma|s, t)}^{S\times S}, \text{ for all } x.$$

This equation has a unique solution for the row vector $\overbrace{\bar{U}(x, \sigma, s)}^{1\times S}$ if and only if the matrix $\overbrace{g(\sigma|s, t)}^{S\times S}$ is non-singular, which is a condition that holds generically.[6]

More generally, σ and t might be restricted to finite sets of different sizes, $\{1, ..., S\}$ and $\{1, ..., T\}$. Perhaps the plausible alternative case is where the number of signals is less than the number of states, and so $S > T$. That is, there are more unknowns, as in $\overbrace{\bar{U}(x, \sigma, s)}^{1\times S}$, than there are equations, where there is one for each signal, t. If the matrix $\overbrace{g(\sigma|s, t)}^{S\times T}$ has full rank, T, then there is again no problem of existence; rather there is an issue of multiplicity—there are many solutions for $\overbrace{\bar{U}(x, \sigma, s)}^{1\times S}$.

We have then proved a simple but illuminating result—

Proposition 1 *Suppose $S \geq T$. If the matrix $\overbrace{g(\sigma|s, t)}^{S\times T}$ has full rank, T, there exists a solution for the row vector $\overbrace{\bar{U}(x, \sigma, s)}^{1\times S}$.*

That is, under the stated conditions, there is a utility function that fully compensates for arbitrary erroneous beliefs on the part of the agent. Hence, despite these beliefs, the first-best evolutionary outcome is attained.

[5] We are grateful to Phil Reny for suggesting the approach adopted in this section.

[6] Generically, any $S \times S$ matrix (with non-negative entries) has a nonzero determinant. Now normalize each column by dividing by the sum of the entries in that column to obtain $\overbrace{g(\sigma|t)}^{S\times S}$. This normalization does not affect the sign of the determinant.

The approach we take in the remainder of the paper reverts to a continuous formulation, but sidesteps the issue of existence by imposing additional structure. This structure is mainly intended to generate a tractable model, and to make available further results, but, as a side effect, it ensures existence.

Comparison with Samuelson and Swinkels

Proposition 1 provides a good vantage point from which to discuss the relationship of the current approach to Samuelson and Swinkels (2006). A crucial difference is the general presumption in Samuelson and Swinkels that the first-best evolutionary outcome is beyond reach, which stands in sharp contrast to Proposition 1. Although the two models have similar motivations, there are many differences in the details. The key difference seems to be as follows. In Samuelson and Swinkels, there is a rich signal structure, with signals lying in $[0,1]^2$. There is not a comparable richness in the utility function. This utility function can be modified by adding a single parameter to encourage one of the two choices that are available to be favored. This utility function does not provide enough flexibility to attain the first-best, given the complexity of the signal structure. However, this inefficiency is less important for them given their emphasis on providing a foundation for behavioral biases.

3 A Monotone Environment

We revert to seeking an optimal utility function of the form $U(x,y,s)$. We first relax the requirement of matching the functions $\mathbb{E}_f\left(\varphi(x,\sigma)\right)$ and $\mathbb{E}_g\left(U(x,y,\sigma)\right)$ asking only that expected utility be maximized by $x^*(s,t)$. That is, we require only that,

$$x^*(s,t) \equiv \arg\max_x \int \varphi(x,\sigma) f(\sigma|s,t)d\sigma = \arg\max_x \int U(x,y,s)g(\sigma|s,t)d\sigma, \text{ for all } s,t.$$

This relaxation of the restrictions on utility is helpful. It is reasonable as well, since there would have been be no biological selection that achieved any more.[7]

Next, we impose assumptions on φ, f, and g. These assumptions guarantee that decisions are "monotone" in the sense that the optimal action is an increasing function of the recurrent and transient signals.

[7] We assume $x^*(s,t)$ is *finite.*

Assumption 1 *i) Fitness* $\varphi(x,\sigma)$ *is twice continuously differentiable and strictly concave in* x.[8]

ii) Increasing the state σ *increases the marginal product of each action:* $\varphi_{x_i\sigma}(x,\sigma) > 0$, *for* $i = 1, ..., N$.

iii) Actions are complements in that $\varphi_{x_i x_j}(x,\sigma) \geq 0$, *for all* $i, j = 1, ..., N, i \neq j$.

Assumption 2 *The pdfs* $f(\sigma|s,t)$ *and* $g(\sigma|s,t)$ *are continuously differentiable in* (s,t) *and strictly increasing, in the sense of first-order stochastic dominance, in* s *and in* t.[9]

These suffice to establish that decisions are monotone—

Lemma 1 *Under Assumptions 1 and 2,* $x_i^*(s,t)$ *is differentiable, with* $\frac{\partial}{\partial s}x_i^*(s,t) > 0$, *and* $\frac{\partial}{\partial t}x_i^*(s,t) > 0$, *for all* i.

Proof. *See the Appendix.* ■

In order to illuminate the current approach, we interweave two examples of economic interest with the development of the theory. The first of these concerns the cost of effort. We consider plausible circumstances under which it would be optimal for evolution to inhibit effort. At the end of this section, we return to this example, giving there evidence on the exact biochemical mechanism by which this inhibition is orchestrated.

Example 1 ***Effort.*** *Suppose* $x \in \mathbb{R}$ *measures costly effort devoted by the individual to a given task, and suppose fitness is given by the net success in this task:*

$$\varphi(x,\sigma) = \sigma x - C(x),$$

where σx *measures a material output (a function of both effort and the random state* σ*), and* $C(x)$ *measures the cost of time and energy in units of fitness, with* $C(x)$ *increasing and convex.*

Given signals s *and* t*, the first-best effort level* $x^*(s,t)$ *equates (expected) marginal benefit and marginal cost:*

$$\mathbb{E}_f\left[\sigma \mid s,t\right] = C'(x).$$

[8] Specifically, we require that the matrix of second derivatives of $\varphi(\cdot,\sigma)$ is everywhere negative definite.

[9] Specifically, we require that $\int v(\sigma)\frac{\partial f(\sigma|s,t)}{\partial s}d\sigma > 0$ for all continuous and strictly increasing functions v; similarly for t.

Assumption 1 is satisfied, so, under Assumption 2, $x^*(s,t)$ is increasing in both signals s and t.

A simple formulation for the cost function C is the familiar quadratic function $C(x) = Ax^2$, which leads to the quadratic fitness function:

$$\varphi(x, \sigma) = \sigma x - Ax^2.$$

The second of these examples concerns the choice of the quality and quantity of offspring. When we return to this example at the end of this section, we argue that it is plausible that utility evolved to compensate for the individual's ignorance of the implications of quality. In the next section, we develop this application further to illustrate how modern behavior might be rendered suboptimal by its reliance on such evolved utility. In particular, we apply the example to help explain the demographic transition as occuring in Europe during the Industrial Revolution.

Example 2 ***Quantity versus quality of offspring.*** *Suppose the individual can have any number of offspring she desires, but the quality of each offspring is determined by a time investment. Ignoring integer constraints on the number of offspring, let $n \in \mathbb{R}_+$ denote the individual's number of offspring and let $x \in \mathbb{R}_+$ denote her time investment per offspring, which we assume to be equal across offspring. Assuming the individual is endowed with one unit of time, she faces the budget constraint*

$$n \cdot x = 1.$$

Suppose fitness is given by

$$n \cdot H(x, \sigma),$$

where H denotes the quality of each offspring, a function of both the time investment x per offspring and the random state σ.

Making use of the budget constraint, fitness can be expressed as function of x and σ alone:

$$\varphi(x, \sigma) = \frac{H(x, \sigma)}{x} \equiv h(x, \sigma).$$

Given signals s and t, the first-best time investment $x^*(s,t)$ equates (expected) average and *marginal quality, which is a familiar result in neoclassical production theory:*

$$\mathbb{E}_f\left[h_x(x^*(s,t),\sigma) \mid s,t\right] = 0 \text{ or } \mathbb{E}_f\left[\frac{H(x^*(s,t),\sigma)}{x^*(s,t)} \mid s,t\right] = \mathbb{E}_f\left[H_x(x^*(s,t),\sigma) \mid s,t\right].$$

Note that, under Assumptions 1, as applied to $h(x,\sigma)$, and 2, the first-best time investment $x^*(s,t)$ is increasing (and the optimal number of offspring is decreasing) in both signals s and t.

A simple formulation for the quality function H that satisfies Assumption 1 is the S-shaped function $H(x,\sigma) = \sigma x^2 - Ax^3$, which again leads to the quadratic fitness function

$$\varphi(x,\sigma) = \frac{H(x,\sigma)}{x} = \sigma x - Ax^2.$$

The Main Result in the Monotone Environment

In order to set the stage for this result, consider an arbitrary $x \in \mathbb{R}^N$. For each s, we associate to the component x_i the value of $t \in \mathbb{R}$ such that the ith component of $x^*(s,t)$ is x_i.

Definition 1 *Let $t^i : \mathbb{R} \times \mathbb{R} \to \mathbb{R}$ be such that $x_i^*(s, t^i(x_i, s)) \equiv x_i$, for all i.*

We now demonstrate the existence of a simple utility function which is uniquely maximized at the optimal x.

Theorem 1 *Under Assumption 1, for all s,t the following utility function implements $x^*(s,t)$—*

$$U(x,y,s) = y + \alpha(x,s).$$

The "adjustment term" is

$$\alpha(x,s) = \sum_i \alpha^i(x_i,s)$$

where

$$\alpha^i(x_i,s) = -\int_0^{x_i}\int \varphi_{x_i}(x^*(s,t^i(z,s)),\sigma)g(\sigma|s,t^i(z,s))d\sigma dz.^{10}$$

Proof. See Appendix. ■

[10] The lower limit in the outer integral is taken to be 0 to ensure convergence.

This is the simplest utility function that delivers $x^*(s,t)$, in that the adjustment term $\alpha(x,s)$ is deterministic (independent of σ), additively separable from y, and additively separable across the components of x. Note also that such an adjustment term must depend on both x and s and thus cannot be further simplified.

Proof in One Dimension

When x is one-dimensional, so $N = 1$, the individual's first-order condition becomes

$$\int \varphi_x(x,\sigma)g(\sigma|s,t)d\sigma - \int \varphi_x(x^*(s,t(x,s)),\sigma)g(\sigma|s,t(x,s))d\sigma = 0.$$

Since $t(x,s)$ is the value of t that induces x as the solution to the constrained optimum, it follows that $x^*(s,t(x,s)) = x$. Hence if $x = x^*(s,t)$ then $t(x,s) = t$ and this first-order condition is satisfied. Further, if $x < x^*(s,t)$, then $t(x,s) < t$. Since $g(\sigma|s,t)$ is increasing in t in the sense of first-order stochastic dominance, it follows that $\int \varphi_x(x,\sigma)g(\sigma|s,t(x,s))d\sigma > \int \varphi_x(x,\sigma)g(\sigma|s,t)d\sigma$, so that marginal expected utility of x is positive. Similarly, if $x > x^*(s,t)$, then marginal expected utility is negative. Hence $x = x^*(s,t)$ is the global maximizer of expected utility.

Sketch of Proof in Two Dimensions

The case in which x is two-dimensional serves to demonstrate the intuitive idea of the general proof, although the general case is more complex technically. With $N = 2$, the first-order conditions for maximizing expected utility are

$$\int \varphi_{x_i}(x,\sigma)g(\sigma|s,t)d\sigma = \int \varphi_{x_i}(x^*(s,t^i(x_i,s)),\sigma)g(\sigma|s,t^i(x_i,s))d\sigma \text{ for } i = 1,2.$$

As required, these first-order conditions are satisfied by $x = x^*(s,t)$ since this implies $t^i(x_i,s) = t$, for $i = 1,2$.

Moreover, $x = x^*(s,t)$ is the unique global maximum. To see why, consider any $x \neq x^*(s,t)$. Figure 1 describes the directions in which expected utility unambiguously increases. These directions can be established by signing the corresponding derivatives. These directions lie in the NE quadrant and the SW quadrant relative to $x^*(s,t)$. There are two representative cases to consider. Case i) $x \geq x^*(s,t)$. In this case, Figure 2 sketches how it is possible to move from a given such x^A to $x^*(s,t)$ in a fashion that increases expected utility. That is, first reduce the coordinate that is too large relative to being on the $x^*(s,\cdot)$

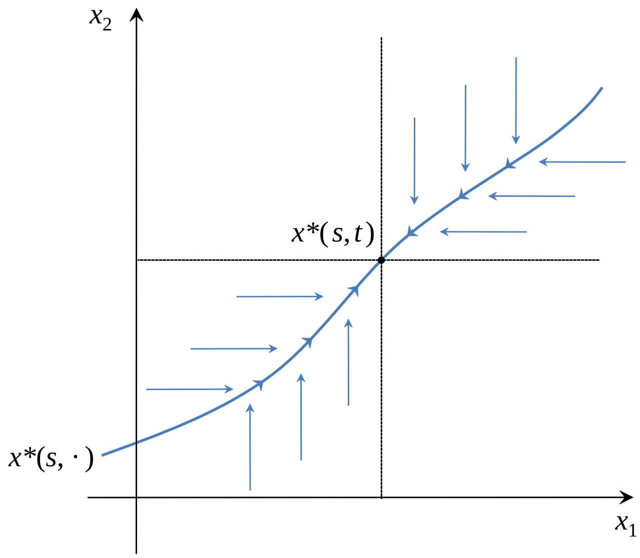

Figure 1: Directions of utility increase

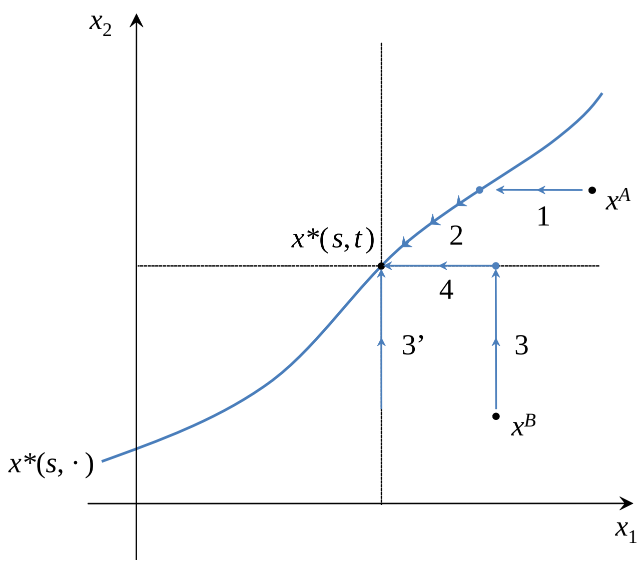

Figure 2: Global optimality of $x^*(s,t)$

curve (step 1). Then move along this curve $x^*(s,\cdot)$ to $x^*(s,t)$ (step 2). (The case in the SW quadrant where $x \leq x^*(s,t)$ is analogous.) Case ii) $x_1 \geq x_1^*(s,t)$ and $x_2 < x_2^*(s,t)$. Refer again to Figure 2. Consider a path from a given such x^B to $x^*(s,t)$ that first increases x_2^B to $x_2^*(s,t)$, as in step 3, and then reduces x_1^B to $x_1^*(s,t)$, as in step 4. Step 4 is a limiting case from Figure 1 where expected utility must increase, but step 3 is apparently ambiguous. Consider, however, step 3', where x_2^B increases to $x_2^*(s,t)$, while holding $x_1 = x_1^*(s,t)$. Expected utility must increase in step 3' since it is again a limiting case from Figure 1. The assumption that $\varphi_{x_1 x_2}(x,\sigma) \geq 0$ implies that expected utility must increase by at least as much in step 3 as it does in step 3', so it must increase in the two-step process—first step 3 and then step 4. The case in which x lies in the NW quadrant is analogous, so the sketch of the proof is complete.

Observations

Note that this particular decomposition of utility generates a particular trade-off between y and x, so the individual would sacrifice expected offspring for, say, more food. Furthermore, this decomposition into y and x is unique, within the additively separable class in which utility cannot depend directly on σ, even though y is itself a function of x as $y = \varphi(x,\sigma)$.[11]

We illustrate the derivation of the adjustment term using Examples 1 and 2:

Example 1 (revisited) Consider now a situation in which the individual's beliefs, as given by the pdf $g(\sigma|s,t)$, strictly first order stochastically dominate the true distribution, as given by the pdf $f(\sigma|s,t)$. It follows that the adjustment term in this case satisfies

$$\alpha_x(x,s) = C'(x) - \mathbb{E}_g(\sigma|s,t(x,s)) < C'(x) - \mathbb{E}_f(\sigma|s,t(x,s)) = 0,$$

since $x = x^*(s,t(x,s))$. That is, the adjustment term acts unambiguously to discourage effort x. Such a situation might arise after illness, when the immune system is depleted, but the individual is not fully cognizant of that fact. In this case, the value of s signals sickness rather than good health. (As health and sickness are recurrent events, utility

[11] A measure of the contribution of Nature to the Individual's decision is

$$\left(\int \varphi_{x_i}(x^*(s,t),\sigma) g(\sigma|s,t) d\sigma\right)^2.$$

This measure is expressed purely in terms of the fitness function and so is independent of the utility representation. It is a measure of how much the optimal choice of $x^*(s,t)$ involves "adjusting" the Individual's preferences away from expected fitness, generating then derivatives of expected offspring that differ from zero.

has been appropriately shaped for either contingency.) It is optimal for there to be less effort in this circumstance, but the individual must be induced to withdraw effort by a term in utility. Indeed, it is believed that, after illness, cytokines are released that inhibit effort, with the individual exhibiting "anhedonia." Evidence for the inhibitory role of cytokines comes from experiments that involve injecting cytokines into healthy rats, which make rats expend less effort for a given reward (see Trivers, 2011). More generally, it may that the individual perceives a cost of effort that varies with the task, even though the actual fitness cost of effort is invariant, as may be the case when comparing walking on a familiar path with walking on a path while visiting a new city.

Example 2 (revisited) It is plausible here that the individual underestimates the importance of quality relative to the importance of quantity. This is because the quantity of offspring is more directly and immediately observable than is their quality, which only shows up as the quantity of distant descendants. Suppose specifically that the individual's beliefs are distorted, with the distribution given by $g(\sigma|s,t)$ being strictly first order stochastically dominated by the true distribution, given by $f(\sigma|s,t)$. Further, the impact of these distorted beliefs is that the individual underestimates the expected marginal effect of the time investment x on average quality, h, so that

$$\alpha_x(x,s) = -\int h_x(x,\sigma)\, g(\sigma|s,t(x,s))\, d\sigma > -\int h_x(x,\sigma)\, f(\sigma|s,t(x,s))\, d\sigma = 0,$$

since $x = x^*(s,t(x,s))$.[12] Now the adjustment term acts to unambiguously encourage the action x, thereby discouraging quantity n and encouraging quality H. We return to this example in the next section, using it to illuminate how the persistence of the adjustment term in the modern environment helps explain the demographic transition.

4 Decisions in the Modern Environment

There are vast differences between the ancestral environment in which our basic preferences evolved and the present environment. A central, though informal, claim in the literature on evolutionary psychology is that such differences have led to "misalignments" in our preferences in the sense that we frequently make fitness-reducing choices. Notably, the preference misalignments studied thus far belong to a single category: those originating in technological

[12] Assumption 1 implies that $h_{xx}(x,\sigma) < 0$, $h_\sigma(x,\sigma) > 0$ and $h_{x\sigma}(x,\sigma) > 0$. We also assume $H_x(x,\sigma) > 0$.

advances that have increased both the general availability of resources and our control over reproduction. Common examples cited in the literature include the tendency of modern humans to consume excessive amounts of sugar and fat and their tendency to use contraceptives. In terms of our model, such misalignments would readily arise upon altering the individual's fitness function while holding his beliefs and utility function constant.

In this section, we investigate a different (and, to our knowledge, novel) category of preference misalignments: those originating from our improved understanding, relative to our hunter-gather ancestors, of the link between actions and fitness. In our model, as we illustrate below, such misalignments arise when changing the individual's beliefs while holding her fitness and utility functions constant. Such misalignments generate a set of testable, revealed preference predictions of the model.

For concreteness, we consider the problem of a perfectly-knowledgeable individual who understands that the state σ is distributed according to f, not g, and is nevertheless endowed with an "ancestral" utility function.[13] In this exercise, to isolate the impact of improved beliefs, we assume that the fitness function φ is the same for the ancestral and modern environments.

Specifically, consider a monotone environment with $N = 1$. Moreover, suppose the individual is endowed with the utility function in Theorem 1:

$$U(x, y, s) = y + \alpha(x, s),$$

where $\alpha(x, s) = -\int_0^x \int \varphi_x(z, \sigma) g(\sigma|s, t(z, s)) d\sigma dz$. Define $x^M(s, t)$ as the solution to the problem of maximizing expected utility $\mathbb{E}_f (U(x, y, s))$, which uses modern beliefs.

Lemma 2 *Impose Assumptions 1 and 2. Fix s and suppose the distribution given by the pdf $f(\sigma \mid s, t)$ is FOSD over that given by $g(\sigma \mid s, t)$ (resp. $g(\sigma \mid s, t)$ is FOSD over $f(\sigma \mid s, t)$) for all t. Then, assuming that $x^M(s, t)$ is finite, $x^M(s, t) > x^*(s, t)$ (resp. $x^M(s, t) < x^*(s, t)$) for all t.*

Proof. Recall that

$$\int \varphi_x(x^*(s, t), \sigma) f(\sigma|s, t) d\sigma = 0 \text{ for all } s, t.$$

[13] More generally, analogous but reduced effects would arise if beliefs g moved in the direction of f in some suitable sense.

Further, for this one dimensional case, since $f(\sigma \mid s,t)$ dominates $g(\sigma \mid s,t)$,

$$\alpha_x(x,s) = -\int \varphi_x(x,\sigma)\, g(\sigma|s,t(x,s))\, d\sigma > -\int \varphi_x(x,\sigma)\, f(\sigma|s,t(x,s))\, d\sigma = 0, \text{ for all } x,s.$$

Hence the first derivative of modern expected utility becomes:

$$\int \varphi_x(x,\sigma) f(\sigma \mid s,t) d\sigma + \alpha'(x,s) > 0, \text{ for all } x \leq x^*(s,t).$$

It follows, as long as a finite solution exists, that $x^M(s,t) > x^*(s,t)$.[14] The proof if $g(\sigma \mid s,t)$ dominates $f(\sigma \mid s,t)$ is analogous. ■

This lemma indicates that when the individual's ancestors underestimated (overestimated) the marginal fitness value of a given action, this individual will select, once perfectly informed, an excessively large (small) action relative to its fitness-maximizing level.

Application to the Demographic Transition

The defining features of the demographic transition, as experienced in Europe in the nineteeth century, are usually taken to be—

- There is first a fall in the mortality rate, often partly due to simple and cheap, but effective, measures to improve public health and medicine. This fall induces a growth spurt in the population.

- The fertility rate eventually falls, reestablishing a rough balance between the birth and death rates.

Additional salient features include:

- Income rises on average during the transition.

- Migration from the country to the city takes place, with a shift away from agriculture and into manufacturing.

- The educational system expands.

[14] If a finite solution is not assumed to exist, it may be that $\int \varphi_x(x,\sigma) dF(\sigma \mid s,t) + \alpha_x(x,s) > 0$, for all x.

The basic puzzle that then arises is: Why did fertility fall sharply following a sharp fall in mortality? That is, given that incomes rose, why would individuals not use the extra resources to produce more offspring?[15]

We show that Example 2 contributes to an explanation of this puzzle—

Example 2 (revisited once more) We illustrate the results by means of an application of the model of the quantity and quality of offspring to the demographic transition. Utility including the adjustment term is now

$$U(x, y, s) = y + \alpha(x, s)$$

where

$$y = h(x, \sigma) \text{ and } \alpha(x, s) = -\int_0^x \int h_x(z, \sigma)\, g(\sigma|s, t(z, s))\, dz d\sigma.$$

If the individual has correct beliefs in the modern environment, it follows that

$$\mathbb{E}_f\left(U_x(x, h(x, \sigma), s\right) = \int h_x(x, \sigma) f(\sigma|s, t)\, d\sigma + \alpha_x(x, s).$$

In order to derive the distortion that arises, consider any x that is no greater than the evolutionarily optimal choice, so that $x \leq x^*(s, t)$. Then, since $\int h_x(x, \sigma) f(\sigma|s, t)\, d\sigma \geq 0$, it follows that $\mathbb{E}_f\left(U_x(x, h(x, \sigma), s\right) + \alpha_x(x, s) > 0$, for such $x \leq x^*(s, t)$. Hence the individual is induced to increase her investment in the quality of offspring beyond the point of evolutionary optimality, correspondingly lowering the quantity of offspring. Thus a shift towards greater appreciation of the merits of the quality of offspring might lead to an overemphasis on quality, at the expense of quantity. Such an effect could then help explain a basic puzzle of the demographic transition. That is, if a concomitant of the transition were greater scientific awareness in general and greater awareness of the consequences of quality in particular, this would lead to an overemphasis on quality, since preferences were already shaped to encourage quality.

This result is reminiscent of the central finding in the multitasking agency literature (Holmstrom and Milgrom, 1991). In both cases the agent's incentive to perform well along one dimension (quantity, here) has the tendency to crowd out her effort along another dimension (quality, here). In the multitasking literature this result emerges from effort along

[15] A compendious detailed reference is Chesnais (1992).

one dimension being relatively easier to assess by the principal. In contrast, in the present setting it arises from the agent's misinformation about the relative value of effort along each dimension. Another difference vis a vis the multitasking literature is that effort is fully contractable here, and thus the first best is attainable by means of a suitable utility adjustment.

We return to the demographic transition at the end of the next section, in which we discuss how imitation might be similarly exaggerated in modern circumstances. In the final section, we return to the demographic transition for the last time, with an integrated model that incorporates imitation that has similarly recently been exaggerated.

5 Imitation, Prepared Learning and the Demographic Transition

In the present section, we show how a relative consumption effect can be generated by the current framework. We achieve this by means of an model that is extended to allow an infinite sequence of individuals, each of whom observes the choice of a predecessor, and to allow the state to evolve. We also make simplifying assumptions—that fitness is quadratic and that the signals are normally distributed.[16] This approach sheds light on an interesting psychological phenomenon—that of "prepared learning". In addition, this approach provides a foundation for the model of Cavalli-Sforza and Feldman (1981) of the demographic transition.

The distortion created by the inaccuracy of individual beliefs is subtle—it arises from the individual's misperception of the informativeness of the transient signal relative to that of the action of the predecessor. Once utility evolved to make a suitable correction for these inaccurate beliefs, it would induce overshooting in a modern environment. That is, individuals might well now imitate too much, to the detriment of evolutionary success.

The key antecedent in this context is Samuelson (2004). The conceptual outline of Samuelson's model is similar—he also supposes that the observable but inherently irrelevant consumption of others conveys information about an unobservable but relevant state. Given the agent's defective statistical assessment of the available signals, it is optimal for evolution to warp the agent's utility function by including relative consumption. The novelty of our approach is in providing a biological basis for utility functions in general. Moreover, as evolutionary explanations for a concern with relative consumption, our models differ in their

[16] Simple versions of Examples 1 and 2 also entail quadratic fitness and normal signals.

details, and, more importantly, we provide new applications to prepared learning and to the demographic transition.

In order to set the stage and to define a suitable notion of "peer effect", consider two individuals who choose sequentially, with individual 1 choosing in the light of her transient signal, t_1, and individual 2 choosing in the light of his transient signal, t_2 and individual 1's choice, x_1. (We will shortly generalize this model to consider an infinite sequence of individuals.) For simplicity, we omit the recurrent signal, s. In the end, the choice of a previous individual will play a role similar to that once played by s. The transient signals are independent, so observing x_1 is useful to individual 2. Given the appropriate utility function, individual 1 effectively maximizes $\int \varphi(x_1, \sigma) f(\sigma|t_1) d\sigma$ by choosing $x_1^*(t_1)$. Given Assumption 1, a key property is that $\frac{dx_1^*(t_1)}{dt_1} > 0$, so individual 2 can infer t_1 from any possible observed value of x_1. We can then formulate individual 2's beliefs and choices in terms of x_1.

Consider now individual 2's ideal choice, $x_2^*(x_1, t_2)$, given x_1, and t_2. This is the choice that maximizes $\int \varphi(x_2, \sigma) \hat{f}(\sigma|x_1, t_2) d\sigma$. In this expression, $\hat{f}$ represents the true pdf for σ conditional on x_1 and t_2. Assume that increases in either of x_1 or t_2 increase the distribution for σ in the sense of first-order stochastic dominance. The problem facing individual 2 remains analogous to that described in detail in Section 4, with x_1 playing the role that was played by s and t_2 playing the role of t.

Applying Theorem 1 in this context, it follows that there exists a utility function whose expectation under individual 2's beliefs, given by the pdf $g(\sigma|x_1, t_2)$, is uniquely maximized by $x_2^*(x_1, t_2)$ of the form

$$U_2(x_2, y_2, x_1) = y_2 + \beta(x_2, x_1).$$

The following captures a "conformity" or "anticonformity" effect—

Definition 2 *Define the (marginal) peer effect as* $\frac{\partial^2 \beta(x_2, x_1)}{\partial x_2 \partial x_1}$. *If this peer effect is positive—* $\frac{\partial^2 \beta(x_2, x_1)}{\partial x_2 \partial x_1} > 0$, *an increase in the action taken by individual 1 increases the marginal utility of the action taken by individual 2, thus spurring an increase in* x_1, *and there is a conformity effect. If this peer effect is negative—*$\frac{\partial^2 \beta(x_2, x_1)}{\partial x_2 \partial x_1} < 0$, *there is an anticonformity effect .*

Extension to Many Individuals

With just two individuals, there must be an asymmetry—one player sees what the other chooses, but there is no-one for that individual to observe. We turn now an infinite sequence of individuals, where all of these, except the first, will be in a position to observe a choice made by a predecessor. This example is also richer in that it allows the state to evolve as a random walk. If there were a fixed state and each individual drew an independent signal concerning this, the uncertainty concerning the state would disappear in the limit, conditional on all the signals. The introduction of a state that evolves as a random walk is also inherently plausible and hence of independent interest. It captures in a stylized fashion how the climate, for example, might evolve over time and therefore lead to a need to continuously update choices.

Suppose individuals $n = 1, 2, ...$ choose in sequence. The state, σ_n, affecting individual n, is a random walk given by $\sigma_{n+1} = \sigma_n + \varepsilon_{\sigma_{n+1}}$ where $\varepsilon_{\sigma_{n+1}} \sim N(0, v)$, for $v > 0$ and $n \geq 1$. At $n = 1$, suppose $\sigma_1 = \varepsilon_{\sigma_1}$. (How the process is initialized is not important in the long run.) Each individual n gets a transient signal $t_n = \sigma_n + \varepsilon_{t_n}$, where $\varepsilon_{t_n} \sim N(0, u)$, and, if $n > 1$, also sees the predecessor's choice x_{n-1}. The random variables ε_{σ_n} and ε_{t_n} are independent of each other and over all n. Fitness for each individual n is given by $-(\sigma_n - x_n)^2$.

Suppose x_n is the optimal choice for individual $n > 1$ given x_{n-1} and t_n. (We write x_n instead of x_n^* for simplicity. The x_n will be characterized below. For $n = 1$, x_1 is optimal given t_1 alone.) Let $\varepsilon_{x_n} = x_n - \sigma_n$ and let $w_n = E(\sigma_n - x_n)^2 = E(\varepsilon_{x_n})^2$, for all n. These denote the error made by individual made by n and its variance, respectively. We can derive a difference equation for the w_n, given by a function ρ, and establish the properties of ρ, as follows:

Lemma 3 *In the peer imitation model, $w_{n+1} = \frac{u(v+w_n)}{u+v+w_n} \equiv \rho(w_n)$, for all n. This difference equation has a unique fixed point $w^* \in (0, u)$ such that $w^* = \rho(w^*)$. Further, the difference equation is stable, with $w_1 = u$ and $w_n \downarrow w^*$, as $n \to \infty$.*

Proof. See Appendix. ■

Consider now the steady state of the above process, which is characterized as follows, dropping the asterisk from w^* for simplicity,

$$x_{n+1} = kt_{n+1} + (1 - k)x_n, \text{ where } k = \frac{v + w}{u + v + w}.$$

It follows then that

$$\frac{\partial k}{\partial u} < 0 \text{ and } \frac{\partial k}{\partial v} > 0.$$[17]

[17]Indeed, $\frac{\partial k}{\partial u} = -\frac{v+w}{(u+v+w)^2} + \frac{u}{(u+v+w)^2}\frac{\partial w}{\partial u}$ and $\frac{\partial k}{\partial v} = \frac{u}{(u+v+w)^2} + \frac{u}{(u+v+w)^2}\frac{\partial w}{\partial v}$. In addition, $\frac{\partial w}{\partial u} = \frac{v}{v+2w} > 0$

Prepared Learning

Consider some relevant results from the psychology literature on prepared learning. Monkeys exhibit neither an inborn fear of snakes nor one of flowers. However, they readily learn to be afraid of snakes if they observe another monkey acting fearfully in the presence of a snake. It is much more difficult to teach them similarly to be afraid of flowers (Cook and Mineka, 1989, for example).

The model sheds light on these phenomena. Suppose that v, the noisiness of the random walk describing the state, decreases (or that u, the noisiness of the transient signal, increases). Since $\frac{\partial k}{\partial v} > 0$ (and $\frac{\partial k}{\partial u} < 0$) it follows that the weight $1-k$ put on the previous individual's choice increases. That is, individual $n+1$ is *more* responsive to individual n's choice. It seems plausible that the evolutionary consequences of snakes have been subject to less drift over time (or that the evolutionary consequences of snakes are less precisely reflected in the transient signal). This is consistent with the observation that individuals are *more* influenced by peer choices concerning snakes than they are by those concerning flowers.

Evolved Utility

The above analysis concerns only what is evolutionarily best and has, so far, abstracted from the beliefs of the individual. Such evolutionarily optimal behavior might be orchestrated despite inaccurate beliefs by means of a suitable utility function. Suppose that the individual's beliefs are given by a pdf g as follows. The individual believes that $t_n = \sigma_n + \varepsilon'_{t_n}$ where $\varepsilon'_{t_n} \sim N(0, u')$ and believes that $x_n = \sigma_{n+1} + \delta_n$ where $\delta_n \sim N(0, z')$. (In contrast, the true steady state distributions are $t_n = \sigma_n + \varepsilon_{t_n}$ with $\varepsilon_{t_n} \sim N(0, u)$ and $x_n = \sigma_{n+1} + \varepsilon_{x_n} - \varepsilon_{\sigma_{n+1}}$ so that $\varepsilon_{x_n} - \varepsilon_{\sigma_{n+1}} \sim N(0, z)$, where $z = v + w$.) Evolved utility, including the adjustment term, is then

$$U(x_n, y_n, x_{n-1}) = y_n + \beta(x_n, x_{n-1}) = -(x_n - \sigma_n)^2 - \frac{k' - k}{k}(x_n - x_{n-1})^2 \text{ where } k' = \frac{z'}{u' + z'}.$$

Indeed, $\frac{\partial E_g U(x_n, y_n, x_{n-1})}{\partial x_n} = 0$ at $x_n = kt_n + (1-k)x_{n-1}$ as is evolutionarily optimal.[18] Thus, because each individual does not see the appropriate relevance of x_{n-1} as a signal of the

and $\frac{\partial w}{\partial v} = \frac{u-w}{v+2w} > 0$, since $u > w$, so that $\frac{\partial k}{\partial u} = \frac{\frac{uv}{v+2w} - (v+w)}{(u+v+w)^2} < 0$ and $\frac{\partial k}{\partial v} > 0$, since $\frac{uv}{v+2w} - (v+w) < 0$ follows from $w = \frac{u(v+w)}{u+v+w}$.

[18] That is, $\frac{\partial E_g U(x_n, y_n, x_{n-1})}{\partial x_n} = -2(x_n - k't_n - (1-k')x_{n-1}) - 2\frac{k'-k}{k}(x_n - x_{n-1}) = 0$ at $x_n = kt_n + (1-k)x_{n-1}$.

state, they are programmed to treat x_{n-1} as an argument of utility. If $k' > k$, it follows that $\frac{\partial^2 \beta(x_n, x_{n-1})}{\partial x_n \partial x_{n-1}} > 0$ so there is then a conformity effect. If $k' > k$, there is an anticonformity effect.

Excessive Mimicry

What would happen if the individual became fully aware of the relevance of both signals? Suppose, that is, that the beliefs g are replaced by the true distribution f, while the utility function remains the same. The existence of the term $\beta(x_n, x_{n-1})$ in utility now leads to a distorted outcome. We now have

$$x_n = \frac{k^2}{k'} t_n + (1 - \frac{k^2}{k'}) x_{n-1}.$$ [19]

It follows that i) If $k' > k$, a fully informed individual would put too much weight on the actions of another. ii) If $k = k'$, then there is no distortion. iii) If $k' < k$, then a fully informed individual puts too little weight on the actions of another.[20]

If it is accepted as a stylized fact that modern individuals have an intrinsic tendency to conform, the relevant case is i). Modern individuals would then weight the actions of others too highly, relative to what would be evolutionarily optimal.

This suboptimality can be captured more precisely as follows. We have $x_{n+1} = \hat{k} t_{n+1} + (1 - \hat{k}) x_n$, for $\hat{k} = \frac{k^2}{k'}$, implying that $x_{n+1} = \sigma_{n+1} + \hat{k} \varepsilon_{t_{n+1}} + (1 - \hat{k})(\hat{\varepsilon}_{x_n} - \varepsilon_{\sigma_{n+1}})$, where $\hat{\varepsilon}_{x_n} = x_n - \sigma_n$ is the random error associated with $\hat{k}$. Let $E\left(\hat{\varepsilon}_{x_n}\right)^2 = \hat{w}$ be the associated steady state error variance. If w^* is the optimal (minimal) variance, then $w^* < \hat{w}$.[21] (Indeed, any procedure that combines the two signals t_{n+1} and x_n in a different linear way than does the optimal procedure generates strictly lower fitness.) Furthermore, the distortion for

[19] This follows since, for this value of x_n,

$$\frac{\partial E_f U(x_n, y_n, x_{n-1})}{\partial x_n} = -2(x_n - k t_n - (1-k) x_{n-1}) - 2\left(\frac{k' - k}{k}\right)(x_n - x_{n-1}) = 0.$$

[20] If $k' \in [k^2, k)$, then $0 \leq 1 - \frac{k^2}{k'} < 1 - k$, and a fully informed individual puts a weight that is too small, but still non-negative, on the actions of the predecessor. If $k' < k^2$ then $0 > 1 - \frac{k^2}{k'}$ so the individual places a *negative* weight on the action of the predecessor.

[21] To prove this, note first that $\hat{w} = \hat{k}^2 u + (1-\hat{k})^2 (v + \hat{w})$. Define $\Delta(w) = w - \hat{k}^2 u + (1-\hat{k})^2 (v + w)$, so that $\Delta(\hat{w}) = 0$, and $\Delta'(w) = 1 - (1-\hat{k})^2 > 0$. It is easily shown that $k^2 u + (1-k)^2 (v+w)$ is *minimized* by choice of $k = \frac{v+w}{u+v+w}$. It follows then that $\Delta(w^*) = w^* - \hat{k}^2 u + (1-\hat{k})^2 (v + w^*) < w^* - k^2 u + (1-k)^2 (v + w^*) = 0$, so $w^* < \hat{w}$, as claimed.

modern choices induced in this way can be relatively arbitrarily large.[22]

5.1 The Demographic Transition

We now apply the imitation model of the current section to add an important component to our treatment of the demographic transition. The imitation model already supports the approach of Cavalli-Sforza and Feldman (1981). They assume that individuals have an intrinsic interest in mimicking the fertility decisions of others and that the cultural transmission of reduced fertility might outweigh natural selection in favor of maintaining high fertility. Our imitation model illustrates how such an interest in mimicry might have evolved. It then shows how a plausible shift in circumstances (better informed individuals) will indeed lead to mimicry being excessive relative to the evolutionary optimum.

We now blend our imitation model with Example 2. For this purpose, we assume that the choice, x, in the imitation model represents the time investment in the quality of each offspring. We also assume that fitness is quadratic. Note that assuming that $\varphi(x,\sigma) = -(\sigma - x)^2$ is operationally equivalent to assuming it is of the form $\varphi(x,\sigma) = h(x,\sigma) = \sigma x - Ax^2$, as in the simple case of Example 2.[23] In the imitation model, there are two signals available to the individual—the contemporaneous transient signal and the action of the predecessor. Suppose now that each individual has beliefs that are erroneous not merely because she misestimates the relative precision of the two signals, but because she underestimates the mean of the state given either signal, as in Example 2 (“ revisited once more”). That is, the individual's beliefs, g, are now given as:

$$t_n = \sigma_n + c + \varepsilon'_{t_n}, \text{ where } \varepsilon'_{t_n} \sim N(0,u') \text{ and } x_n = \sigma_{n+1} + d + \delta_n \text{ where } \delta_n \sim N(0,z').$$

All other elements of the imitation model remain as before. The relative precision of the predecessor's choice is underestimated, so that $k' = \frac{z'}{u'+z'} > k = \frac{z}{u+z}$. To capture the underestimation of the state given the signals, we further assume that $c, d > 0$.

[22] To prove this, consider the limit as $v \to 0$. From $w = \frac{u(v+w)}{u+v+w}$ and $k = \frac{v+w}{u+v+w}$, it follows that both $w \to 0$ and $k \to 0$. If $k' > 0$ is fixed, then $\hat{k} = \frac{k^2}{k'} \to 0$ as well. From $\hat{w} = \hat{k}^2 u + (1-\hat{k})^2(v+\hat{w})$, it follows that $\hat{w} = \frac{\hat{k}u}{2-\hat{k}} + \frac{(1-\hat{k})^2 v}{(2-\hat{k})\hat{k}}$. Using $w = \frac{u(v+w)}{u+v+w}$ and $w = ku$, it follows that $1 = \frac{u}{\frac{v}{k^2}} + k$, so that $\frac{v}{k^2} \to u$ as $v \to 0$. Hence $\hat{w} \to \frac{uk'}{2} > 0$, but $w^* \to 0$, as $v \to 0$,as claimed.

[23] Indeed, $-(\sigma - x)^2 = -\sigma^2 + 2x\sigma - x^2 = 2(x\sigma - \frac{x^2}{2}) - \sigma^2$. Although the term $-\sigma^2$ varies with the pdf, it has no effect on the optimal choice, x.

In the ancient steady state, evolved utility of the form

$$U(x_n, y_n, x_{n-1}) = y_n + \beta(x_n, x_{n-1}) = -(x_n - \sigma_n)^2 - \frac{k' - k}{k}(x_n - x_{n-1})^2 + 2\,(k'c + (1 - k')d)\,x_n$$

would have generated evolutionary optimality.[24]

Consider now the modern distorted choice that arises with this utility function, but with the correct beliefs, f. From the first-order condition $\frac{\partial E_f U(x_n, y_n, x_{n-1})}{\partial x_n} = 0$, it follows that

$$x_n = \frac{k^2}{k'}t_n + (1 - \frac{k^2}{k'})x_{n-1} + \frac{(k'c + (1 - k')d)\,k}{k'}.$$

We now derive the increased variance associated with this choice of x_n. The effect of underestimating of the mean of the state is to add a new term to this variance. Fomally, let $x_n - \sigma_n = \hat{\varepsilon}_{x_n}$, in the steady state. Suppose $E\,(x_n - \sigma_n) = \hat{m}$ and $E\,(x_n - \sigma_n - \hat{m})^2 = \hat{w}$. It follows that $\hat{m} = \frac{k'c+(1-k')d}{k}$ and the equation that determines $\hat{w}$ is $\hat{w} = \hat{k}^2 u + (1 - \hat{k})^2(v + \hat{w})$, as above when $c = d = 0$.[25] Finally, we have that $E\,(x_n - \sigma_n)^2 = \hat{w} + (\hat{m})^2$, as claimed.

Combined Impact of Evolved Concerns for Imitation and for Offspring Quality

To summarize: Not only does the individual place too much weight on the choice of the predecessor, but she also uniformly increases her own choice. The first effect causes the variance of the choice to be too high, and so lowers fitness. The second effect independently raises the mean of the choice and therefore also raises the variance and further lowers fitness.

The central mystery of the demographic transition concerns the fall in fertility that occurred despite, or because of, a rise in income. This juxtaposition is puzzling even from an economic point of view, but it can be resolved by supposing a suitably high level of concern with the quality of offspring (see Becker and Lewis, 1973, for example). This juxtaposition is still more challenging to explain as evolutionarily optimal.

[24] To see why, note that the first-order condition

$$\frac{\partial E_g U(x_n, y_n, x_{n-1})}{\partial x_n} = -2(x_n - k't_n - 2(1 - k')x_{n-1} - \gamma) - \frac{k' - k}{k}(x_n - x_{n-1}) + 2(k'c + (1 - k')d) = 0,$$

implies $x_n = kt_n + (1 - k)x_{n-1}$.

[25] These results follow from the steady state relationship

$$x_n = \sigma_n + \hat{k}\varepsilon_{t_n} + (1 - \hat{k})(\hat{\varepsilon}_{x_{n-1}} - \varepsilon_{\sigma_n}) + \frac{(k'c + (1 - k')d)k}{k'},$$

where $\hat{k} = \frac{k^2}{k'}$.

We finesse this challenge by arguing that our evolved utility functions might well have led to evolutionary suboptimality in the circumstances of the transition. That is, a tendency to imitate and a concern with quality were embedded in utility by misperceptions that were plausibly once prevalent. If these misperceptions were reduced at the time of the transition, the result would be overimitation combined with an exaggerated concern with quality, thus at once buttressing the approach of Cavalli-Sforza and Feldman (1981) and that of Becker and Lewis (1973).

6 Conclusions

Our motivating question was: Supposing, for the sake of argument, that we are intelligent and rational, why is our evolved utility not simply offspring? We formulated a principal-agent model in which both Nature and the individual observed two signals—one recurrent and one transient—that bear on the fitness consequences of the agent's choices. The agent, however, has arbitrary beliefs about the implications of the signals. One abstract option would be for Nature to explicitly and directly communicate her accurate beliefs to the agent, who could then choose optimally by maximizing expected fitness in the light of these beliefs. This option, however, is simply not a good description of humans, even though we are presumably closer than any other species.

Alternatively, we consider the option that Nature shapes the utility function in the light of the recurrent signal only. The individual then maximizes the expectation of this utility conditional on her arbitrary beliefs. We show, remarkably, that this option could also generate optimal choice in the context of the model. That this method seems to be one in evidence, despite the existence of a theoretically more direct way of achieving the same end, may then have been harmless phylogenetic happenstance.

We showed that utility is a "whispering within" urging individuals to take actions that reflect the evolutionary wisdom of a multitude of ancestors, in addition to accounting for local on-the-spot information. The loudness of the whisper, or the force of the push delivered by Nature, ultimately derives from the extent to which the individual's beliefs were erroneous.

In vastly changed modern conditions the mechanism would no longer be evolutionarily optimal. Indeed, if the individual now has fully accurate beliefs, modern decisions will be distorted in accordance with the loudness of the whisper. In these circumstances, once utility has arguments other than offspring, it will generate systematically biased choices, providing empirical content for the current theory.

7 Appendix

7.1 Proof of Lemma 1.

The function $x^*(s,t)$ is characterized by the first-order conditions

$$\int \varphi_{x_i}(x^*(s,t),\sigma)f(\sigma|s,t)d\sigma = 0, \text{ for } i = 1, ..., N.$$

Hence

$$\sum_j A_{ij} \frac{\partial x_j^*(s,t)}{\partial t} = b_i \text{ for } i = 1, ..., N,$$

where

$$A_{ij} = \int \varphi_{x_i x_j}(x^*(s,t),\sigma)f(\sigma|s,t)d\sigma \text{ and } b_i = -\int \varphi_{x_i}(x^*(s,t),\sigma)\frac{\partial f(\sigma|s,t)}{\partial t}d\sigma < 0.$$

The $n \times n$ matrix A is symmetric, negative definite, and has non-negative off-diagonal elements. Hence $-A$ is a Stieltjes matrix, which must have a symmetric and non-negative inverse (see Varga, (1962, p. 85). Hence A^{-1} must be a symmetric and non-positive matrix. Since

$$\begin{bmatrix} \frac{\partial x_1^*(s,t)}{\partial t} \\ \cdots \\ \frac{\partial x_N^*(s,t)}{\partial t} \end{bmatrix} = A^{-1}b,$$

it follows that $\frac{\partial x_j^*(s,t)}{\partial t} \geq 0$, for $j = 1, ..., N$. Further, since A^{-1} is non-singular, it cannot have any row be entirely zero, and it must indeed be that $\frac{\partial x_j^*(s,t)}{\partial t} > 0$, for $j = 1, ..., N$.

The proof that $\frac{\partial x_j^*(s,t)}{\partial s} > 0$, for $j = 1, ..., N$ is analogous.

7.2 Proof of Theorem 1

Select an arbitrary s. To simplify notation, we then drop the dependence of $x^*(\cdot)$ and $\alpha(\cdot)$ on s. Define, for all x and t,

$$V(x;t) = \mathbb{E}\left[\varphi(x,\sigma) \mid t\right] + \alpha(x),$$

where the expectation is taken over σ using the pdf g.

We wish to show that $V(x^*(t), t) > V(x, t)$ for all t and all $x \neq x^*(t)$.

Remark 1 *Properties of $V(x;t)$. For all i and all t:*

1. $\frac{\partial}{\partial x_i} V(x_i, x_{-i}; t)$ *is weakly increasing in* x_{-i} *for all* x_i.
2. $\frac{\partial}{\partial x_i} V(x; t)$ *is strictly increasing in* t *for all* x.
3. $\frac{\partial}{\partial x_i} V(x^*(t); t) = 0$.

Proof. From the definitions of $V(\cdot)$ and $\alpha(\cdot)$ we obtain

$$\frac{\partial}{\partial x_i} V(x; t) = \mathbb{E}\left[\varphi_{x_i}(x_i, x_{-i}, \sigma) \mid t\right] - \mathbb{E}\left[\varphi_{x_i}(x^*(t^i(x_i)), \sigma) \mid t^i(x_i)\right]. \tag{3}$$

For property 1, note that the first term on the R.H.S. of (3) is weakly increasing in x_{-i} (since, by assumption, $\frac{\partial^2}{\partial x_i \partial x_j} \varphi(x, \sigma) \geq 0$ for all x, σ and all $i \neq j$), and the second term is independent of x_{-i}.

For property 2, note that the first term on the R.H.S. of (3) is increasing in t (since, by assumption, $\frac{\partial^2}{\partial x_i \partial \sigma} \varphi(x, \sigma) > 0$ for all x, σ and all i, and the pdf g is increasing in t in first-order stochastic dominance), and the second term is independent of t.

For property 3, note that $t^i(x_i^*(t)) = t$ (by definition) and therefore

$$\mathbb{E}\left[\varphi_{x_i}(x^*(t), \sigma) \mid t\right] = \mathbb{E}\left[\varphi_{x_i}(x^*(t^i(x_i)), \sigma) \mid t^i(x_i)\right].$$

■

Now select an arbitrary t and an arbitrary $x \neq x^*(t)$. Let $\tau_i = t^i(x_i)$ for all i. Assume, WLOG, that $\tau_1 \leq \tau_2 \leq ... \leq \tau_N$. Also, select two numbers τ_0 and τ_{N+1} such that $\tau_0 \leq \min\{\tau_1, t\}$ and $\tau_{N+1} \geq \max\{\tau_N, t\}$.

Define

$$\begin{aligned} M_+ &= \{i : x_i \geq x_i^*(t)\}, \\ M_- &= \{i : x_i < x_i^*(t)\}. \end{aligned}$$

Note that

$$V(x \vee x^*(t); t) =$$
$$V(x; t) + \sum_{n \in M_-} \int_{\tau_n}^{\min\{\tau_{n+1}, t\}} \frac{d}{d\tau} V(x^*_{i \leq n}(\tau), x_{i>n}; t) d\tau,$$

and

$$V(x \vee x^*(t); t) =$$
$$V(x^*(t); t) + \sum_{n \in M_+} \int_{\max\{\tau_{n-1}, t\}}^{\tau_n} \frac{d}{d\tau} V(x^*_{i \in M_-}(t), x_{i \in M_+^n}, x^*_{i \geq n}(\tau); t) d\tau,$$

where M_+^n is defined as the set $\{i \in M_+ : i < n\}$.

It follows that

$$V(x^*(t); t) - V(x; t) = \sum_{n \in M_-} \int_{\tau_n}^{\min\{\tau_{n+1}, t\}} \frac{d}{d\tau} V(x^*_{i \leq n}(\tau), x_{i>n}; t) d\tau \tag{4}$$
$$- \sum_{n \in M_+} \int_{\max\{\tau_{n-1}, t\}}^{\tau_n} \frac{d}{d\tau} V(x^*_{i \in M_-}(t), x_{i \in M_+^n}, x^*_{i \geq n}(\tau); t) d\tau.$$

We begin by showing that $V(x^*(t); t) \geq V(x; t)$, for which we proceed in two steps.

Step 1. We show that all terms in the first sum of (4) are nonnegative. Fix $n \in M_-$. For all $\tau \in (\tau_n, \min\{\tau_{n+1}, t\})$ (a possibly empty interval) we have

$$\frac{d}{d\tau} V(x^*_{i \leq n}(\tau), x_{i>n}; t) = \tag{5}$$
$$\sum_{j \leq n} \frac{\partial}{\partial x_j} V(x^*_{i \leq n}(\tau), x_{i>n}; t) \cdot \frac{d}{d\tau} x^*_j(\tau)$$
$$\geq \sum_{j \leq n} \frac{\partial}{\partial x_j} V(x^*_{i \leq n}(\tau), x_{i>n}(\min\{\tau_{n+1}, t\}); t) \cdot \frac{d}{d\tau} x^*_j(\tau)$$
$$\geq \sum_{j \leq n} \frac{\partial}{\partial x_j} V(x^*_{i \leq n}(\tau), x_{i>n}(\min\{\tau_{n+1}, t\}); \min\{\tau_{n+1}, t\}) \cdot \frac{d}{d\tau} x^*_j(\tau) > 0.$$

(Recall that $\frac{d}{d\tau} x^*_j(\tau) > 0$ for all j.)

The first weak inequality in (5) follows from property 1 of the remark: $x_i \geq x_i(\min\{\tau_{n+1}, t\})$

for all $i > n$ implies

$$\frac{\partial}{\partial x_j} V(x^*_{i\leq n}(\tau), x_{i>n}; t) \geq$$
$$\frac{\partial}{\partial x_j} V(x^*_{i\leq n}(\tau), x_{i>n}(\min\{\tau_{n+1}, t\}); t) \text{ for all } j \leq n.$$

The second weak inequality in (5) follows from property 2 of the remark: $t \geq \min\{\tau_{n+1}, t\}$ implies

$$\frac{\partial}{\partial x_j} V(x^*_{i\leq n}(\tau), x_{i>n}(\min\{\tau_{n+1}, t\}); t) \geq$$
$$\frac{\partial}{\partial x_j} V(x^*_{i\leq n}(\tau), x_{i>n}(\min\{\tau_{n+1}, t\}); \min\{\tau_{n+1}, t\}) \text{ for all } j \leq n.$$

Finally, the strict inequality in (5) follows from combining all three properties of the remark: $\tau < \min\{\tau_{n+1}, t\}$ implies $x^*_i(\tau) < x^*_i(\min\{\tau_{n+1}, t\})$ for all i and therefore

$$\frac{\partial}{\partial x_j} V(x^*_{i\leq n}(\tau), x_{i>n}(\min\{\tau_{n+1}, t\}); \min\{\tau_{n+1}, t\}) \geq$$
$$\frac{\partial}{\partial x_j} V(x^*(\tau); \min\{\tau_{n+1}, t\}) > \frac{\partial}{\partial x_j} V(x^*(\tau); \tau) = 0 \text{ for all } j \leq n.$$

Step 2. We show that all terms in the second sum of (4) are nonpositive. Fix $n \in M_+$. Note that for all $\tau \in (\max\{\tau_{n-1}, t\}, \tau_n)$ (a possibly empty interval) we have

$$\begin{aligned} &\frac{d}{d\tau} V(x^*_{i\in M_-}(t), x_{i\in M^n_+}, x^*_{i\geq n}(\tau); t) = \\ &\sum_{j\geq n} \frac{\partial}{\partial x_j} V(x^*_{i\in M_-}(t), x_{i\in M^n_+}, x^*_{i\geq n}(\tau); t) \cdot \frac{d}{d\tau} x^*_j(\tau) \\ &\leq \sum_{j\geq n} \frac{\partial}{\partial x_j} V(x^*_{i<n}(\max\{\tau_{n-1}, t\}), x^*_{i\geq n}(\tau); t) \cdot \frac{d}{d\tau} x^*_j(\tau) \\ &\leq \sum_{j\geq n} \frac{\partial}{\partial x_j} V(x^*_{i<n}(\max\{\tau_{n-1}, t\}), x^*_{i\geq n}(\tau); \max\{\tau_{n-1}, t\}) \cdot \frac{d}{d\tau} x^*_j(\tau) < 0. \end{aligned} \tag{6}$$

The first weak inequality in (6) follows from property 1 of the remark: $\left(x^*_{i\in M_-}(t), x_{i\in M^n_+}\right) \leq$

$x^*_{i<n}(\max\{\tau_{n-1},t\})$ implies

$$\begin{aligned}\frac{\partial}{\partial x_j}V(x^*_{i\in M_-}(t),x_{i\in M_+^n},x^*_{i\geq n}(\tau);t) &\leq \\ \frac{\partial}{\partial x_j}V(x^*_{i<n}(\max\{\tau_{n-1},t\}),x^*_{i\geq n}(\tau);t) \text{ for all } j &\geq n.\end{aligned}$$

The second weak inequality in (6) follows from property 2 of the remark: $t \leq \max\{\tau_{n-1},t\}$ implies

$$\frac{\partial}{\partial x_j}V(x^*_{i<n}(\max\{\tau_{n-1},t\}),x^*_{i\geq n}(\tau);t) \leq$$
$$\frac{\partial}{\partial x_j}V(x^*_{i<n}(\max\{\tau_{n-1},t\}),x^*_{i\geq n}(\tau);\max\{\tau_{n-1},t\}) \text{ for all } j \geq n.$$

Finally, the strict inequality in (6) follows from combining all three properties of the remark: $\tau > \max\{\tau_{n-1},t\}$ implies $x^*_i(\tau) > x^*_i(\max\{\tau_{n-1},t\})$ for all i and therefore

$$\frac{\partial}{\partial x_j}V(x^*_{i<n}(\max\{\tau_{n-1},t\}),x^*_{i\geq n}(\tau);\max\{\tau_{n-1},t\}) \leq$$
$$\frac{\partial}{\partial x_j}V(x^*(\tau);\max\{\tau_{n-1},t\}) < \frac{\partial}{\partial x_j}V(x^*(\tau);\tau) = 0 \text{ for all } j \geq n.$$

We now show that $V(x^*(t);t) > V(x;t)$. Since $x \neq x^*(t)$ there exists either an $n \in M_-$ such that the interval $(\tau_n, \min\{\tau_{n+1},t\})$ is nonempty, or an $n \in M_+$ such that the interval $(\max\{\tau_{n-1},t\},\tau_n)$ is nonempty (or both). In the former case, it follows from step 1 above that at least one of the integrals in the first sum of (4) is positive. In the latter case, it follows from step 2 above that at least one of the integrals in the second sum of (4) is negative. This completes the proof of Theorem 1.

7.3 Proof of Lemma 3

As part of the inductive proof, we show that ε_{x_n} is independent of ε_{σ_m} and ε_{t_m} for $m = n+1, n+2, ..,$ and for $n = 1, 2, ...$ At $t = 1$, it follows that $\sigma_1 = t_1 - \varepsilon_{t_1}$ so it follows that the optimal $x_1 = t_1$. (Whenever fitness is quadratic, the evolutionarily optimal choice of x is the mean of the distribution of σ.) Furthermore, it follows that $w_1 \equiv E(\sigma_1 - x_1)^2 = E(\varepsilon_{t_1})^2 = u$, where $x_1 - \sigma_1$ is independent of ε_{σ_n} and ε_{t_n} for $n = 2, ...$. As the induction hypothesis, suppose the result holds for n so that, in particular, ε_{x_n} is independent of ε_{σ_m} and ε_{t_m} for

$m = n+1, n+2, \ldots$ It follows that the posterior distribution of σ_{n+1} given x_n and t_{n+1} is normal with mean given by

$$k_n t_{n+1} + (1-k_n)x_n \text{ where } k_n = \frac{v + w_n}{u + v + w_n} = k(u, v, w_n),$$

and variance given by

$$w_{n+1} = k_n^2 u + (1-k_n)^2 (v + w_n) = \frac{u(v + w_n)}{u + v + w_n} = \rho(w_n),$$

say. The optimal choice of x_{n+1} is then $x_{n+1} = k_n t_{n+1} + (1-k_n)x_n$ conditional on t_{n+1} and x_n, given the quadratic fitness function.[26] Now, completing part of the proof, we have $\varepsilon_{x_{n+1}} = x_{n+1} - \sigma_{n+1}$ is independent of ε_{σ_m} and ε_{t_m} for $m = n+2, \ldots$.

Then $\rho'(w_n) = (1-k_n)^2 \in (0,1)$. Also $\rho(0) = \frac{uv}{u+v} > 0$ and $\rho(u) = \frac{(u+v)u}{2u+v} < u$. It follows that the difference equation $w_{n+1} = \rho(w_n)$ generates monotonic convergence $w_n \downarrow w^*$, as $n \to \infty$ from $w_1 = u$ to the unique $w^* \in (0, u)$ such that $w^* = \rho(w^*)$.

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