

# Network Externalities and Market Dominance\*

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## Abstract

We develop a simple framework in the spirit of Katz and Shapiro (1985) and Becker (1991) to study pricing and competition in markets with network externalities. We provide micro-foundations for a family of non-standard “in/out” demand curves, with a positively-sloped intermediate region. We also provide a simple theory of equilibrium selection that yields concise predictions regarding when firms will be “in” and when “out.” We use this framework to provide a unified explanation for a variety of stylized facts (some already accounted for, others not) including why: (i) it is difficult to become “in”; (ii) the “in” position is fragile; (iii) “in” firms are not asleep in the sense that they continuously raise quality and keep prices low; and (iv) “in” firms acquire startups in order to entrench their position.

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# 1 Introduction

Recent decades have witnessed the rise of “superstar firms” that manage to take over most of their markets (e.g. Autor et al. (2017)). Frequently, these firms produce goods that are both scalable (such as non-rival goods) and subject to network effects whereby consumer demand is directly affected by peer consumption. For example, *Facebook*, *Microsoft*, *Google*, and *Uber* all produce goods with essentially zero marginal costs and whose consumers are arguably influenced by the overall popularity of the product in question.<sup>1</sup>

Commonly, markets with scalable goods and network effects exhibit a series of interrelated patterns:

First, winners serve a disproportionate share of their market, with a large size gap between them and their closest rivals.

Secondly, it is difficult to become a winner and yet success is so fragile that it can vanish overnight. The difficulties of becoming popular (going from being “out” to being “in”) are illustrated by Microsoft’s search engine *Bing*, which despite years of sizeable investments remains much smaller than the current superstar *Google*. And the ease with which a successful firm can suddenly fail (going from “in” to “out”) is illustrated by the web browser *Netscape*, which despite its initial dominant status was overtaken quite suddenly by Microsoft’s *Internet Explorer*.

Finally, despite their seemingly dominant position, winners are not asleep. This can be seen by the fact that winners tend to continuously raise quality (for example, by purchasing new startups) while at the same time keeping their prices low, sometimes even below average cost.

Our goal is to develop a tractable framework in the spirit of Katz and Shapiro (1985) and Becker (1991) that provides a unified account for these patterns. In addition, we use this framework to characterize the role and economic value of

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<sup>1</sup> In 2002, Bill Gates summarized his ambitions as follows: “We look for opportunities with network externalities — where there are advantages to the vast majority of consumers to share a common standard. We look for businesses where we can garner large market shares, not just 30-35%” (Rivkin and Van den Steen (2009)).

“influencers” (consumers capable of affecting a sizeable fraction of the population), to discuss investment and firm acquisition decisions, and to argue that when network externalities are strong, antitrust considerations differ significantly from conventional approaches.

Katz and Shapiro (1985) and Becker (1991) first studied the role of network externalities in models where peer consumption enters as a direct argument of a consumer’s demand (in addition to price). They argue that network externalities can generate demand curves with both downward- and upward-sloping regions, depending on when the traditional price effect or the network effect dominates. Such demand curves admit the possibility of multiple equilibria in terms of firm size, and therefore the possibility of large industry winners.

What is somewhat less satisfying in these models is their treatment of equilibrium selection, which is for the most part absent, and their justification for a particular shape for the demand curve. In Katz and Shapiro’s model, the population of consumers is assumed to be uniform; as a result, only through non-linearities in the network externality can one obtain a non-standard demand curve. And in Becker’s model, a non-standard demand with desirable properties is simply assumed. In addition, while Becker argues persuasively that price is a somewhat blunt instrument in affecting a move from the “small” equilibrium to the “large” equilibrium, his argument in the other direction is less convincing.

Our model is closest to Becker’s, but with one ingredient left out, and one added in. We dispense with his capacity constraint (i.e. goods in our model are non-rivalrous), and we model the equilibrium selection or transition process explicitly. In addition, we provide a micro-foundation for a Becker-style demand curve with alternating downward and upward sloping regions, which we call an “in/out” demand curve. This micro-foundation shows that such non-standard shape is quite easy to obtain, even when network externalities are linear, and in fact should be expected to arise frequently in practice.

In our setting consumers may coordinate on a high or low level of demand. This is somewhat analogous to a setting studied in Akerlof and Holden (2017) in which investors can coordinate on a high or low level of investment in a project.

As in Akerlof and Holden (2017) we view the coordinating process as part of the equilibrium and we consider how a firm might be able to coordinate consumers on a high, rather than a low, level of demand.

The remainder of the paper is organized as follows. Section 3 presents the baseline model with a single firm; Section 4 considers price competition among two firms; Section 5 considers the special case of a piecewise linear “In/Out” demand curve; and Section 6 concludes.

## 2 Monopoly Case

To begin, we will consider a setting with a single firm (a monopolist). We assume that each period ( $t = 1, 2, \dots, T$ ), the monopolist chooses a price  $p^t$ . The marginal cost of production is constant — and we normalize it to zero.

There is a set of consumers. Consumer  $i$ 's demand at time  $t$ , denoted  $q_i^t$ , depends upon the price  $p^t$  and upon aggregate consumption  $Q^t$ ; that is,  $q_i^t = D_i(p^t, Q^t)$ . We assume  $q_i^t$  is decreasing in  $p^t$ ; and  $q_i^t$  is increasing in  $Q^t$ , reflecting the presence of positive network externalities.

For any given  $p^t$ , an equilibrium quantity  $Q^{t*}$  satisfies:

$$Q^{t*} = \sum_i D_i(p_t, Q^{t*})$$

We will focus attention on cases where this gives rise to “in/out” aggregate demand curves of the type shown in Figure 1. The idea, loosely speaking, is that when total consumption is low, the network effect is weak and demand has a standard shape (i.e. a negative slope). When total consumption exceeds a first critical level ( $Q_L$  in the figure), the network externality begins to dominate, causing marginal value to grow with quantity. When total consumption exceeds a second critical level ( $Q_H$  in the figure), the network externality is exhausted and demand again has a standard shape. The next section formalizes this possibility.

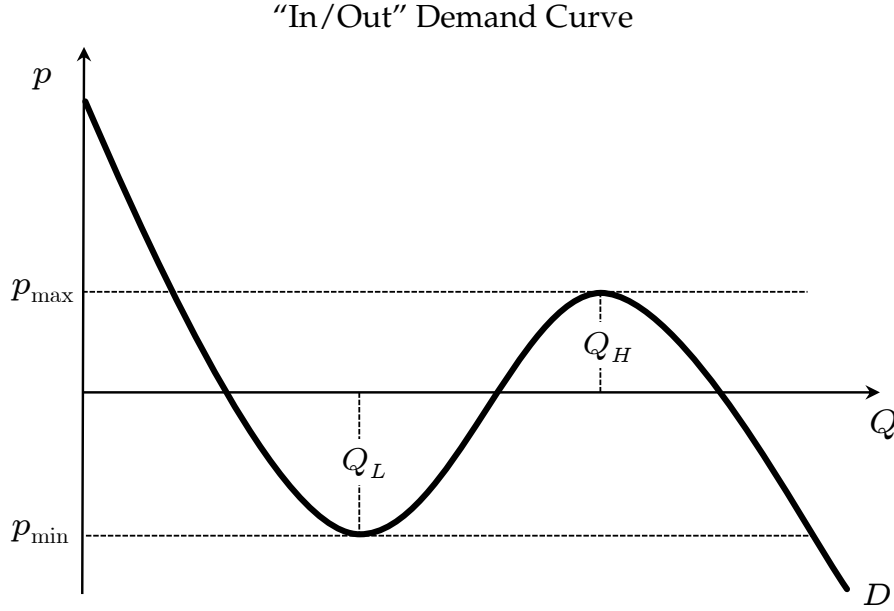


Figure 1

## 2.1 Micro-foundation for In/Out Demand

Assume there is a continuum of consumers ( $i \in [0, 1]$ ). Consumer  $i$  has an idiosyncratic taste  $\theta_i$  for the monopolist's good. The  $\theta_i$ 's are distributed iid according to a distribution with cdf  $F$ .

Consumers face a binary choice whether to consume. We normalize to zero the utility from not consuming; and we assume the utility from consuming is  $\theta_i + \mu + \alpha \cdot Q - p$ , where  $\mu$  is the quality of the monopolist's good relative to the outside option,  $Q$  is the population mass currently consuming, and  $p$  is the price. The parameter  $\alpha$  captures the size of the network externalities.

Under these assumptions, agent  $i$  consumes if and only if  $\theta_i + \mu + \alpha \cdot Q - p \geq 0$ . Therefore, the agents who consume are those whose idiosyncratic taste for the good exceeds a threshold:  $\theta_i \geq \hat{\theta}$ , where

$$\hat{\theta} = \underbrace{p - \mu}_{\text{effective price}} - \underbrace{\alpha \cdot Q}_{\text{network externality}} \quad (1)$$

The threshold is increasing in the good's "effective price" (the price net of quality) and it is decreasing in the size of the network externality.

Observe that aggregate demand  $Q$  is equal to the mass of consumers above the threshold:

$$Q = 1 - F(\hat{\theta}). \quad (2)$$

Combining equations (1) and (2), we find that:

$$Q = 1 - F(p - \mu - \alpha \cdot Q). \quad (3)$$

Rearranging terms, we obtain a formula for (inverse) demand:

$$p^d(Q) = F^{-1}(1 - Q) + \alpha \cdot Q + \mu. \quad (4)$$

Notice from equation (4) that  $\mu$ , the good's quality relative to the outside option, shifts the demand curve vertically. An increase in the good's quality shifts demand vertically up while an increase in the outside option shifts demand vertically down.

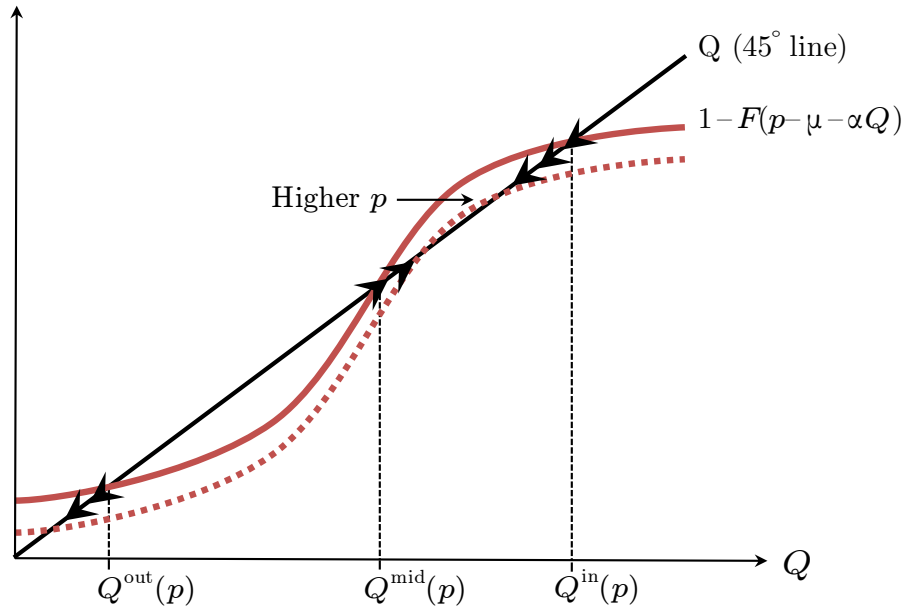
We can differentiate equation (4) to obtain an expression for the slope of demand:

$$\frac{dp^d(Q)}{dQ} = \underbrace{\alpha}_{\text{network externality}} - \underbrace{\frac{1}{f(F^{-1}(1 - Q))}}_{\text{distribution of consumers' tastes}}. \quad (5)$$

The slope depends both upon the size of network externalities (term 1) and the distribution of consumers' tastes (term 2). Demand is downward-sloping in the absence of network externalities; but demand may be positively sloped when network externalities are present.

From here, it is easy to obtain an in/out demand curve. Suppose, for instance,  $F$  is a normal distribution with mean 0 and variance  $\sigma^2$ . The demand curve has a maximum slope of  $\alpha - \frac{1}{f(0)} = \alpha - \sqrt{2\pi}\sigma^2$  (at  $Q = \frac{1}{2}$ ). The demand curve has a minimum slope of  $\alpha - \frac{1}{f(\pm\infty)} = -\infty$  (at  $Q = 0$  and  $Q = 1$ ). Demand has an in/out shape, as in Figure 1, if the network externalities are sufficiently large ( $\alpha >$

(a) Demand curve solves:  $Q = 1 - F(p - \mu - \alpha Q)$



(b) "In/Out" Demand Curve

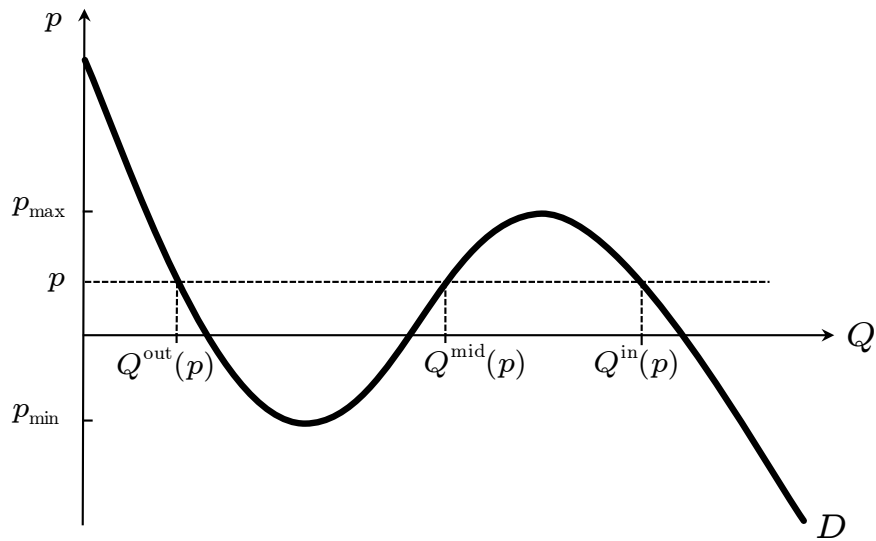


Figure 2

$\sqrt{2\pi\sigma^2})$ .<sup>2</sup>

More generally, if  $F$  has support  $\mathbb{R}$  and the pdf is single-peaked:

1. Demand is downward-sloping if  $\alpha \leq \hat{\alpha}$ .
2. Demand is “in/out” if  $\alpha > \hat{\alpha}$ .

To understand the in/out shape, consider the graphical representation of equation (3) in Figure 2a. Each point along the in/out demand curve is a solution to this equation. Figure 2a shows why, at intermediate prices, there are multiple possible quantities demanded:  $Q^{out}(p)$ ,  $Q^{mid}(p)$ , or  $Q^{in}(p)$ . Figure 2a also shows the impact of a change in price.  $Q^{out}(p)$  and  $Q^{in}(p)$  both decrease when  $p$  rises; correspondingly, the demand curve is downward sloping at  $Q^{out}(p)$  and  $Q^{in}(p)$  (see Figure 2b). In contrast,  $Q^{mid}(p)$  increases when  $p$  rises; correspondingly, the demand curve is upward sloping at  $Q^{mid}(p)$ .

## 2.2 Equilibrium Selection

If the firm sets an intermediate price ( $p_{\min} < p < p_{\max}$ ), there are three intersections with the in/out demand curve — hence three Nash equilibria. We will denote these equilibria as  $Q^{out}(p)$ ,  $Q^{mid}(p)$ , and  $Q^{in}(p)$  (where  $Q^{out}(p) < Q^{mid}(p) < Q^{in}(p)$ ).

Given that there are three Nash equilibria, we will invoke a refinement of Nash equilibrium due to Kets and Sandroni (2015) called “introspective equilibrium.” Introspective equilibrium is based upon level- $k$  thinking (see Crawford et al. (2013) for a survey). Kets and Sandroni assume that agents have exogenously-given “impulses,” which determine how they play at level 0. At level  $k > 0$ , each agent formulates a best response to the belief that opponents are at level  $k - 1$ . Introspective equilibrium is defined as the limit of this process at  $k \rightarrow \infty$ .

We can denote by  $\bar{q}_{ik} \in \{0, 1\}$  agent  $i$ 's decision whether to consume when he is at level- $k$ . We can denote by  $\bar{Q}_k$  the aggregate consumption level at level  $k$ . The following is a precise definition of introspective equilibrium in our setting.

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<sup>2</sup>If consumers' tastes instead follow a uniform distribution, the demand curve has a constant slope because  $f(\theta)$  is constant.



**Definition 1** (Introspective Equilibrium).

1. Fix  $p$  and take consumers' level-0 choices, called "impulses," as given:  $\bar{q}_{i0}$ . The "aggregate impulse" is  $\bar{Q}_0$ .
2. Level  $k$  is obtained by assuming consumers best-respond to price  $p$  and the belief that other consumers are at level  $k - 1$ :  $\bar{q}_{ik}, \bar{Q}_k$ .
3. An introspective equilibrium is the limit as  $k \rightarrow \infty$ :  $q_i^*, Q^*$ .

Introspective equilibrium is very general. It nests a wide range of refinement concepts, corresponding to different assumptions about agents' impulses. Risk dominance, for instance, corresponds to the case where agents are uncertain about each others' impulses. We make what we think is a natural assumption in our setting. We assume each agent's impulse is to do what she did in the previous period:  $\bar{q}_{i0} = q_i^{t-1}$ . Therefore the aggregate impulse is  $\bar{Q}_0 = Q^{t-1}$ . We take agents' impulses in the first period as exogenous and denote them as  $q_i^0$  and  $Q^0$ . Proposition 1 derives the introspective equilibrium as a function of the aggregate impulse ( $Q^{t-1}$ ).

**Proposition 1.** *The unique introspective equilibrium is:*

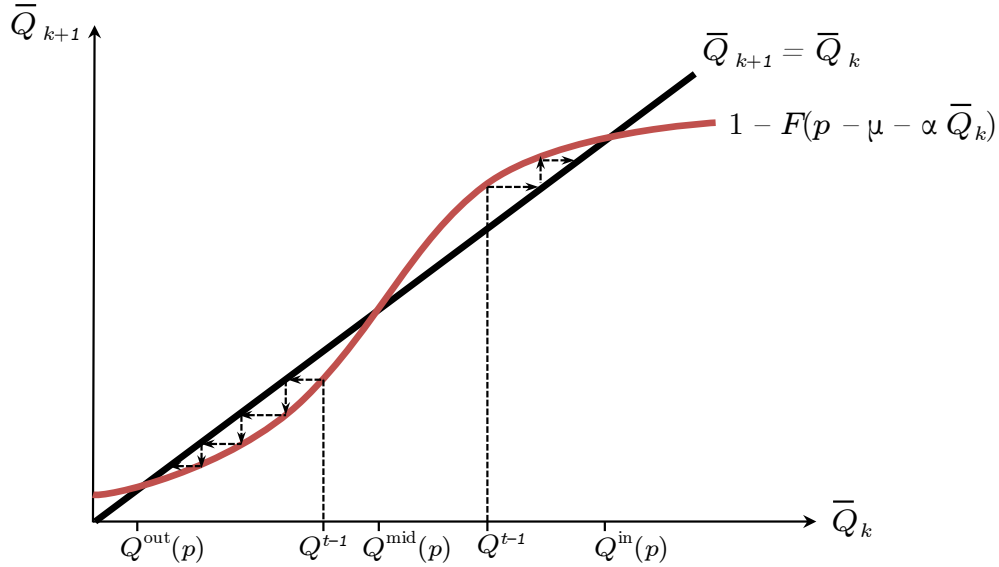
$$Q^*(p, Q^{t-1}) = \begin{cases} Q^{in}(p), & \text{if } Q^{t-1} > Q^{mid}(p). \\ Q^{mid}(p), & \text{if } Q^{t-1} = Q^{mid}(p). \\ Q^{out}(p), & \text{if } Q^{t-1} < Q^{mid}(p). \end{cases}$$

**Proof of Proposition 1**

The proof is easy to derive. From equation (3), we obtain a simple formula for the evolution of aggregate consumption between levels  $k$  and  $k + 1$ :

$$\bar{Q}_{k+1} = 1 - F(p - \mu - \alpha \bar{Q}_k) \tag{6}$$

$$\bar{Q}_{k+1} = 1 - F(p - \mu - \alpha \bar{Q}_k)$$



**Figure 3**

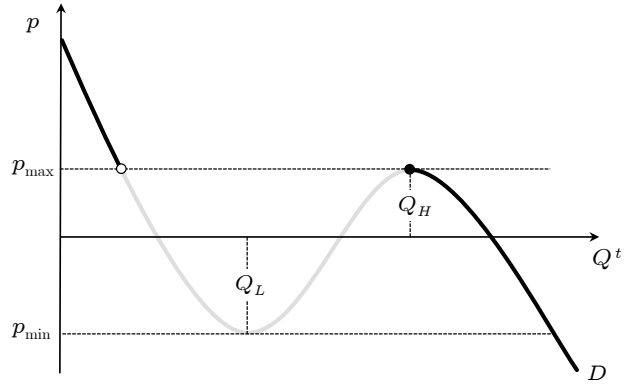
Figure 3 corresponds to equation (6). It shows how, starting from an initial impulse ( $Q^{t-1}$ ), the aggregate consumption level evolves.

Observe that, if there is a high initial impulse to consume ( $Q^{t-1} > Q^{mid}(p)$ ), aggregate consumption increases between levels 0 and 1. Intuitively, the high level of consumption at level 0 drives more agents to consume at level 1. Consumption continues to increase between levels 2 and 3, 3 and 4, and so forth, reaching  $Q^{in}(p)$  in the limit. Hence, when  $Q^{t-1} > Q^{mid}(p)$ , the introspective equilibrium is  $Q^{in}(p)$ . By a similar logic, aggregate consumption falls between successive levels when  $Q^{t-1} < Q^{mid}(p)$ . When  $Q^{t-1} < Q^{mid}(p)$ , the introspective equilibrium is  $Q^{out}(p)$ . Q.E.D.

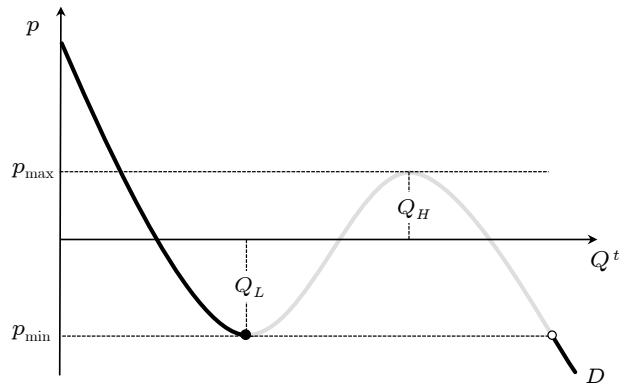
Corollary 1 presents the main result in this section. It tells us that, depending on the impulse, the firm faces one of three negatively-sloped demand curves shown in Figure 4.

**Corollary 1.** *In any period, the firm faces one of three downward-sloping demand curves (depending upon  $Q^{t-1}$ ):*

(a) "In" Demand Curve ( $Q^{t-1} \geq Q_H$ )



(b) "Out" Demand Curve ( $Q^{t-1} \leq Q_L$ )



(c) "Between" Demand Curve ( $Q_L < Q^{t-1} < Q_H$ )

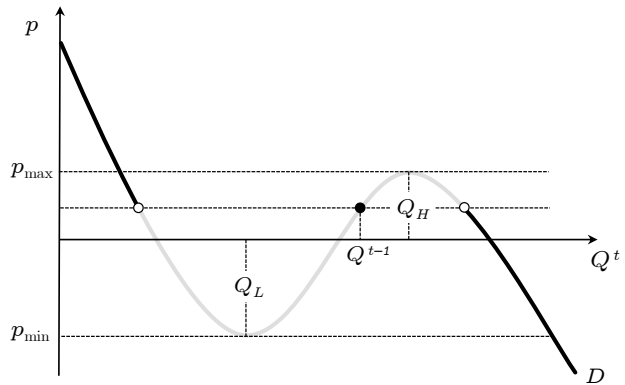


Figure 4

1. “In” Demand Curve ( $Q^{t-1} \geq Q_H$ ).
2. “Out” Demand Curve ( $Q^{t-1} \leq Q_L$ ).
3. “Between” Demand Curve ( $Q_L < Q^{t-1} < Q_H$ ).

If  $Q^{t-1} \geq Q_H$ , the introspective equilibrium is always  $Q^{in}(p)$ . We will say that the firm is “in” when  $Q^{t-1} \geq Q_H$ . The firm, in this case, faces an “in” demand curve (shown in Figure 4a).

If  $Q^{t-1} \leq Q_L$ , the introspective equilibrium is always  $Q^{out}(p)$ . We will say that the firm is “out” when  $Q^{t-1} \leq Q_L$ . The firm, in this case, faces an “out” demand curve (shown in Figure 4b).

If  $Q_L < Q^{t-1} < Q_H$ , we will say that the firm is “between.” The firm, in this case, faces a “between” demand curve (shown in Figure 4c).

## 2.3 Optimal Pricing

We are now in a position to formally state the monopolist’s problem and describe optimal pricing.

In each of the  $T$  periods, the monopolist chooses price to maximize the discounted sum of future profits. The monopolist’s discount factor is  $\delta$ . Period  $t$  profits are  $\pi^t = p^t \cdot Q^t$ , and  $Q^t = Q^*(p^t, Q^{t-1})$  (where  $Q^*(p^t, Q^{t-1})$  is defined as in Proposition 1). We take  $Q^0$ , the initial consumption level or impulse, as given.

First, suppose the firm is myopic ( $\delta = 0$ ), caring only about current profits. There are three cases (see Lemma 1).

**Lemma 1.** *Suppose the firm is myopic ( $\delta = 0$ ). Depending upon the shape of the demand curve (which is a function of  $\alpha$ ,  $\mu$ , and  $F$ ), there are three cases:*

- (i) *It is optimal to go “in” in period  $t$  (choose  $p^t$  such that  $Q^t \geq Q_H$ ) regardless of the value of  $Q^{t-1}$ .*
- (ii) *It is optimal to go “out” in period  $t$  (choose  $p^t$  such that  $Q^t \leq Q_L$ ) regardless of the value of  $Q^{t-1}$ .*

(iii) It is optimal for “in” (“out”) firms to stay “in” (“out”). “Between” firms go “in” (“out”) if  $Q^{t-1}$  is above (below) a cutoff ( $Q^{myopic}$ ).

One implication of Lemma 1 is that firms that are initially “between” ( $Q_L < Q^0 < Q_H$ ) never remain “between”: they either become “in” or “out.” The reason is as follows. To remain between, the firm would need to choose a price  $p^1$  such that  $Q^1 = Q^0$ . But, as Figure 4c shows, starting at such a price, a small decrease in the price increases demand by a discontinuous amount. Thus, the hypothesized price cannot be optimal.

Now, suppose the firm is not myopic ( $\delta > 0$ ). Proposition 2 compares the optimal pricing with that of a myopic firm.

**Proposition 2.** *Suppose the firm is not myopic. In cases (i) and (ii) in Lemma 1, the firm acts as if it were myopic. In case (iii), “out” and “between” firms potentially act non-myopically in period 1. Specifically, if  $Q^0$  exceeds a cutoff ( $Q^{non-myopic} < Q^{myopic}$ ), the firm prices below the myopic level in period 1; it goes “in” and stays “in” in subsequent periods.*

Intuitively, “out” and “between” firms have an incentive to price below the myopic level in order to become “in” in subsequent periods. The benefit of being “in” is that “in” firms face a better demand curve than “out” firms (see Figure 4).

In practice, some firms charge no money to their consumers. Notable examples are search engines (e.g. Google, Bing, Yahoo) and web browsers (e.g. Firefox, Chrome, Safari, Internet Explorer). In these instances, firms profit by showing advertisements to their users and/or by directing user traffic to other products. Since, in the margin, these practices reduce consumer value, they can be interpreted as a positive price.

## 2.4 Influencers

Suppose an “influencer” is connected to a fraction  $\phi$  of the consumers and has the ability to shift those consumers’ impulses: from “don’t consume” to “consume” or vice-versa. Let  $b \in \{0, 1\}$  denote the influencer’s choice whether to shift consumers’ impulses to “consume” or “don’t consume.”

The influencer can play a pivotal role in determining whether the firm is “in” or “out,” as the following corollary to Proposition 1 demonstrates.

**Corollary 2.** *The firm faces*

1. An “In” Demand Curve if:

$$(1 - \phi)Q^{t-1} + \phi b \geq Q_H.$$

2. An “Out” Demand Curve if:

$$(1 - \phi)Q^{t-1} + \phi b \leq Q_L.$$

3. A “Between” Demand Curve otherwise.

Given that influencers can help firms become — or stay — “in,” firms may be willing to pay influencers for their services. An “out” firm can become “in” without help from an influencer by dropping its price below  $p_{min}$  for one period; but this is expensive. With help from an influencer, it may be possible for a “out” firm to become “in” without dropping its price. In a competitive setting, an “in” firm might also pay an influencer in order to protect its “in” position against rivals (see Section 3 for further discussion of the competitive case).

One way in which an influencer might operate is by changing default options. Defaults appear to have the ability to alter consumer expectations — and/or act as nudges — and thereby affect initial impulses.

An illustration can be seen in the “browser war” in the 90s between the initially dominant Netscape and Microsoft’s Internet Explorer. After initial difficulties to penetrate the market, Internet Explorer eventually managed to displace Netscape; and when this happened, it happened suddenly.<sup>3</sup> The key to overtaking Netscape was a deal between Microsoft and the internet provider AOL, whereby AOL agreed to set Internet Explorer as its default browser in exchange for valuable advertising. As Yoffie and Cusumano (1998) note: “To entice Steve Case, the

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<sup>3</sup>In the mid 1990s, Netscape controlled roughly 80% of the market; by the early 2000s, Internet Explorer controlled more than 90%.

CEO of AOL, to make Internet Explorer AOL's preferred browser, Gates offered to put an AOL icon on the Windows 95 desktop, perhaps the most expensive real estate in the world. In exchange for promoting Internet Explorer as its default browser, AOL would have almost equal importance with [AOL's rival] MSN on future versions of Windows." To this day, the browser wars continue, with smartphones being the latest battlefield. Here again, defaults appear to play a major role (e.g. Cain Miller (2012)).

### **Remark: Large Consumers**

Suppose, in addition to small consumers, there is a large consumer who, when he purchases the monopolist's good, purchases a large quantity. Like an influencer, a large consumer can help tip the monopolist from "out" to "in." Consequently, one would expect the monopolist to pay the large consumer a rent — just as he would pay a rent to an influencer. This rent might take the form of a discount relative to the price charged to small consumers.

## **3 Competition**

It is a simple step to move from a monopoly setting to a competitive setting. Suppose there are two firms (1 and 2) that engage in price competition. At stage 1, firm 1 sets price  $p_1$ . At stage 2, firm 2 sets price  $p_2$ .

We continue to assume there is a continuum of consumers ( $i \in [0, 1]$ ) with tastes  $\theta_i$  distributed  $F$ . Now,  $\theta_i$  represents consumer  $i$ 's taste for good 1 *relative to* good 2. Consumers make a binary choice whether to consume good 1 or good 2. Hence, overall demand sums to 1:  $Q_1 + Q_2 = 1$ . The utility from consuming good 1 is  $\theta_i + \mu + \alpha \cdot Q_1 - p_1$ , where  $\mu$  denotes the quality of good 1 relative to good 2. The utility from consuming good 2 is  $\alpha \cdot Q_2 - p_2$ .

Under these assumptions, demand for each good depends upon the price differential:  $\Delta = p_1 - p_2$ . We will focus on cases where this gives rise to in/out demand, as pictured in Figure 5. Observe that, whenever demand for good 1 is

“In/Out Demand Curve”: Price Competition

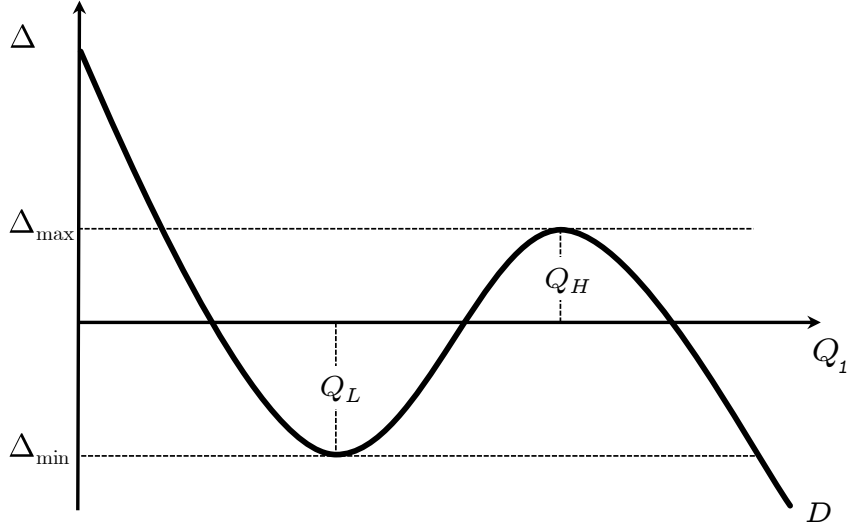


Figure 5

in/out, demand for good 2 will also be in/out (given that  $Q_2 = 1 - Q_1$ ).<sup>4</sup>

We can use the same technique as before to derive a formula for demand. Observe that consumer  $i$  chooses good 1 over good 2 if and only if  $\theta_i + \mu + \alpha \cdot Q_1 - p_1 \geq \alpha \cdot Q_2 - p_2$ . Therefore, the agents who consume good 1 are those whose taste for good 1 exceeds a threshold  $\hat{\theta}$ , where:

$$\begin{aligned} \hat{\theta} &= \underbrace{p_1 - p_2}_{\Delta} - \mu - \alpha \cdot (Q_1 - \underbrace{Q_2}_{1-Q_1}) \\ &= \underbrace{\Delta - \mu}_{\text{effective price}} - \underbrace{\alpha \cdot (2Q_1 - 1)}_{\text{network externality}}. \end{aligned} \quad (7)$$

Given that demand for good 1 consists of the mass of consumers above the threshold:

$$Q_1 = 1 - F(\hat{\theta}). \quad (8)$$

<sup>4</sup>Were we to plot the in/out demand curve for good 2, we would place  $p_2 - p_1 = -\Delta$  on the y-axis rather than  $\Delta$ .



Combining (7) and (8), we obtain an analog of equation (3):

$$Q_1 = 1 - F(\Delta - \mu - \alpha \cdot (2Q_1 - 1)). \quad (9)$$

Rearranging terms yields an expression for the (inverse) demand for good 1:

$$\Delta^d(Q_1) = \mu + \alpha \cdot (2Q_1 - 1) + F^{-1}(1 - Q_1). \quad (10)$$

Observe that a change in the quality of good 1 relative to good 2,  $\mu$ , shifts demand vertically. Differentiating equation (10), we obtain a formula for the slope of demand:

$$\frac{d\Delta^d(Q)_1}{dQ_1} = \underbrace{2\alpha}_{\text{network externality}} - \underbrace{\frac{1}{f(F^{-1}(1 - Q_1))}}_{\text{distribution of consumers' tastes}}. \quad (11)$$

As before, if  $F$  has support  $\mathbb{R}$  and the pdf is single-peaked:

1. Demand is downward-sloping if  $\alpha \leq \hat{\alpha}$ .
2. Demand is “in/out” if  $\alpha > \hat{\alpha}$ .

### **Remark: Product Compatibility**

The existing literature on network externalities has highlighted product compatibility as an issue of interest: particularly, the incentives of firms to make their products compatible (see, for instance, Katz and Shapiro (1994)).

It is easy to incorporate into our framework the idea that competing products may be more or less compatible. Suppose the utility from consuming good 1 is  $\theta_i + \mu + \alpha \cdot (Q_1 + \gamma Q_2) - p_1$  and the utility from consuming good 2 is  $\alpha \cdot (Q_2 + \gamma Q_1) - p_2$ . Parameter  $\gamma \in [0, 1]$  can be thought of as the compatibility of the goods; when the goods are more compatible, the consumers of good 1 derive more utility from the consumption of good 2 (and vice-versa). The baseline model corresponds to the case of perfect incompatibility ( $\gamma = 0$ ).

This addition to the model has the following effect on the threshold for con-

suming good 1:

$$\hat{\theta} = \Delta - \mu - \underbrace{\alpha(1 - \gamma)}_{\text{"effective network parameter"}} \cdot (2Q_1 - 1). \quad (12)$$

The only change relative to the baseline model is that  $\alpha(1 - \gamma)$  appears in place of  $\alpha$ . Hence, greater product compatibility (higher  $\gamma$ ) is equivalent, from the point of view of the firms, to smaller network externalities (lower  $\alpha$ ).

### 3.1 Equilibrium Selection

When the price differential  $\Delta$  is in an intermediate range, there are multiple Nash equilibria ( $Q_1^{out}(\Delta)$ ,  $Q_1^{mid}(\Delta)$ , and  $Q_1^{in}(\Delta)$ ). To select between them, we will use the same equilibrium refinement as in the single-firm case. This yields the following analog of Proposition 1.

**Proposition 3.** *In the unique introspective equilibrium:*

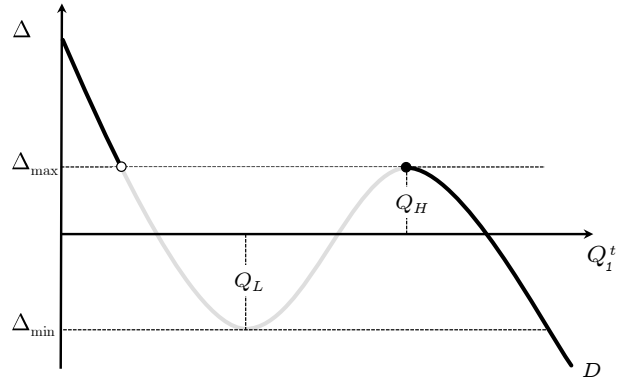
$$Q_1^*(\Delta, Q_1^{t-1}) = \begin{cases} Q_1^{in}(\Delta), & \text{if } Q_1^{t-1} > Q_1^{mid}(\Delta). \\ Q_1^{mid}(\Delta), & \text{if } Q_1^{t-1} = Q_1^{mid}(\Delta). \\ Q_1^{out}(\Delta), & \text{if } Q_1^{t-1} < Q_1^{mid}(\Delta). \end{cases}$$

The following corollary to Proposition 3 is analogous to Corollary 1. It says that firm 1 faces one of the three negatively-sloped demand curves shown in Figure 6.

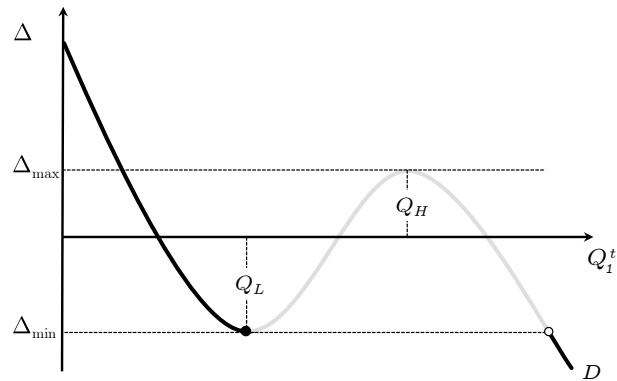
**Corollary 3.** *In any period, firm 1 faces one of three downward-sloping demand curves (depending upon  $Q_1^{t-1}$ ):*

1. "In" Demand Curve ( $Q_1^{t-1} \geq Q_H$ ).
2. "Out" Demand Curve ( $Q_1^{t-1} \leq Q_L$ ).
3. "Between" Demand Curve ( $Q_L < Q_1^{t-1} < Q_H$ ).

(a) "In" Demand Curve: Price Competition ( $Q_1^{t-1} \geq Q_H$ )



(b) "Out" Demand Curve: Price Competition ( $Q_1^{t-1} \leq Q_L$ )



(c) "Between" Demand Curve: Price Competition ( $Q_L < Q_1^{t-1} < Q_H$ )

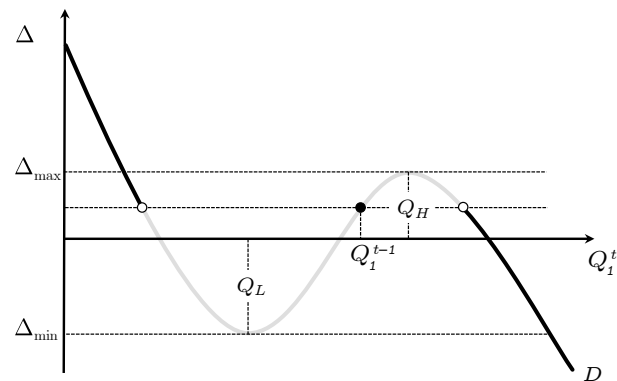


Figure 6

We will again refer to a firm as “in,” “out,” or “between” depending upon whether it faces an “in,” “out,” or “between” demand curve. Because overall demand is fixed ( $Q_1 + Q_2 = 1$ ), either both firms are “between” or one is “in” and the other is “out.”<sup>5</sup>

We will focus attention on the case where firm 1 starts “in” and firm 2 starts “out.” This corresponds to many cases of interest, where competition is between an established firm that has built up a network and a recent entrant.

### 3.2 Analysis

We are now in a position to formally state and analyze the pricing game played by the firms.

At stage 1, firm 1 sets a price  $p_1$ ; at stage 2, firm 2 sets a price  $p_2$ . The resulting payoffs to the firms are  $\pi_1 = p_1 \cdot Q_1^{in}(\Delta)$  and  $\pi_2 = p_2 \cdot (1 - Q_1^{in}(\Delta))$ , where  $\Delta = p_1 - p_2$ . Observe that  $\pi_1$  and  $\pi_2$  depend upon the shape of the demand curve; the demand curve, in turn, depends upon parameters  $\alpha$ ,  $\mu$ , and  $F$ .

We can use backward induction to solve for the equilibrium of the game. Let  $p_2^{BR}(p_1)$  denote firm 2’s best response to price  $p_1$  and let  $\Delta(p_1) = p_1 - p_2^{BR}(p_1)$ . Firm 1 chooses  $p_1$  to maximize:

$$\pi_1(p_1) = p_1 \cdot Q_1^{in}(\Delta(p_1)).$$

Firm 1 “remains in” if  $\Delta(p_1) \leq p_{\max}$  and “falls out” if  $\Delta(p_1) > p_{\max}$ . We will refer to  $\Delta(p_1) \leq p_{\max}$  as the “remain-in constraint” or RIC. Demand for good 1 decreases discontinuously when firm 1 falls out. Hence, there is an incentive for firm 1 to choose a price that satisfies RIC. Furthermore — and most importantly — RIC will be a *binding* constraint in a region of the parameter space.

It is easy to show that, to remain in, firm 1 must set a price below a threshold  $\bar{p}_{\text{RIC}}$  (the argument is given below as part of the proof of Proposition 4). Therefore,

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<sup>5</sup>It might be possible for both firms to be “in” if overall demand were not fixed.

the following is an equivalent formulation of the remain-in constraint:

$$p_1 \leq \bar{p}_{\text{RIC}}(\alpha, \mu, F). \quad (\text{RIC})$$

Proposition 4 characterizes how a change in the goods' relative qualities ( $\mu$ ) affects the equilibrium outcome in the region where RIC binds.

**Proposition 4.** *When RIC binds, increases in the quality of good 1 relative to good 2, as measured by  $\mu$ :*

1. *Translate one-to-one into increases in good 1's equilibrium price:*

$$\frac{\partial \bar{p}_{\text{RIC}}}{\partial \mu} = 1.$$

2. *Have no effect on good 2's equilibrium price.*
3. *Have no effect on equilibrium quantities ( $Q_1$  and  $Q_2$ ).*

#### **Proof of Proposition 4**

Figure 7 shows the demand for good 2 for a particular value of  $p_1$ . Firm 2's best response to  $p_1$  is either to choose:

1. The profit-maximizing price conditional on staying "out" ( $p^{local}$  in the figure).
2. The profit-maximizing price conditional on going "in."

At least when firm 2 is on the margin between staying "out" or going "in,"  $p_{min}$  will be the profit-maximizing price conditional on going "in."

The red region in Figure 7 represents the profits to firm 2 from choosing  $p^{local}$ ; the blue region represents the profits from choosing  $p_{min}$ . Observe that RIC is satisfied when the red region is (weakly) larger than the blue region; RIC binds when the regions are of equal size.

### The Remain-In Constraint (RIC)

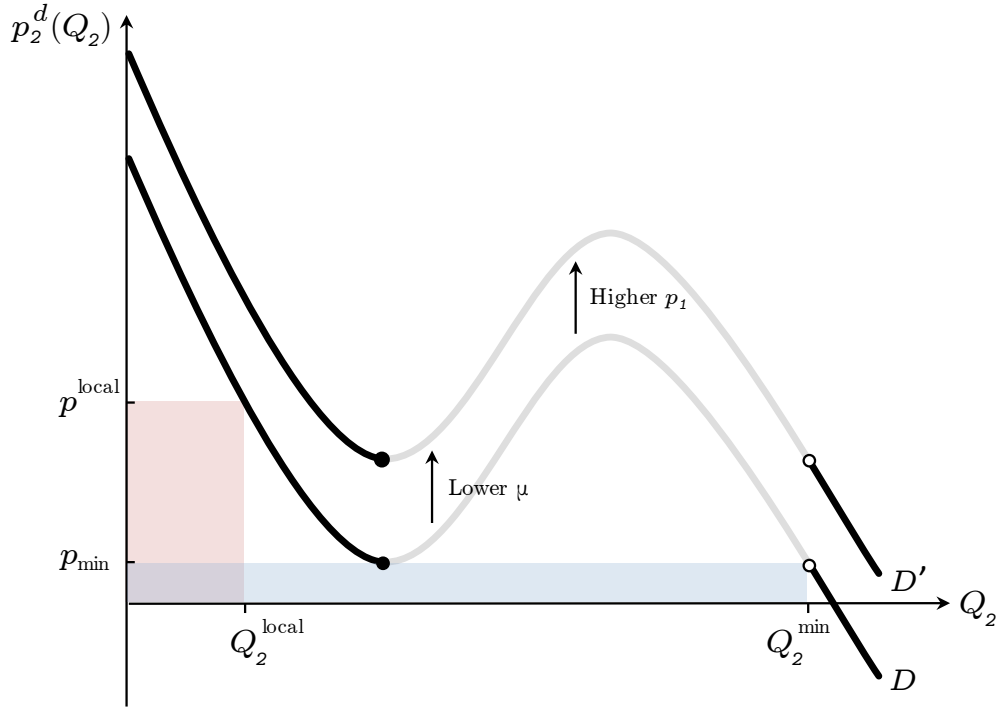


Figure 7

An increase in  $p_1$  shifts firm 2's demand curve vertically up, which increases the size of the blue region relative to the red region.<sup>6</sup> This explains why firm 1 must price below a threshold,  $\bar{p}_{RIC}$ , in order to meet RIC.

Suppose RIC is a binding constraint and suppose demand curve  $D$  in Figure 7 depicts the place where RIC exactly binds. Observe that the demand curve firm 2 faces depends upon the “effective price” of good 1:  $p_1 - \mu$ . Hence, if  $\mu$  decreases by an amount  $\Delta\mu$ , firm 1 must decrease  $p_1$  by  $\Delta\mu$  to stay on demand curve  $D$ . This explains why, in the region where RIC binds, a change in  $\mu$  changes  $p_1$  by an equivalent amount. Furthermore, since firm 2 always faces the same demand curve  $D$  in the region where RIC binds, it always charges the same price ( $p^{local}$ ) and sells the same quantity ( $Q_2^{local}$ ). QED.

In practice, “in” firms may need to charge low — even zero — prices to sat-

<sup>6</sup>The reason the blue region increases in size more than the red region when  $p_1$  increases is that  $Q_2^{min} > Q_2^{local}$ .

isfy the RIC constraint. For example, despite their overwhelming market shares, Google (in web search), Uber (in ride sharing), and Amazon Web Services (in cloud computing) all keep their prices low — arguably to stunt the rise of their nearest rivals.

### 3.3 Incentives for Innovation

The fact that when network externalities are large, the winning and losing firms compete for the “in” position, as opposed to merely competing for a single marginal consumer, has important implications for the firms’ incentives to innovate.

To illustrate, suppose the two firms have an opportunity to invest up front on R&D activities that raise the intrinsic quality of their respective products. Let  $\mu_i$  denote the intrinsic quality of firm’s  $i$  product, with  $\mu = \mu_1 - \mu_2$ . Suppose quality  $\mu_i$  costs  $C(\mu_i)$  to obtain, where  $C$  is twice differentiable and satisfies  $C', C'' > 0$  and  $C'(0) = 0$ . Suppose  $\mu_1$  and  $\mu_2$  are observed by both firms before they engage in price competition. (The exact timing of the choices of  $\mu_1$  and  $\mu_2$  is immaterial.)

Corollary 4 shows that, when the network externality is strong, the two firms face radically different incentives to innovate:

**Corollary 4.** *Consider the extended model with investments. Suppose firm 1 retains the “in” position and suppose that, in the pricing stage, the remain-in constraint is binding (that is, the network externality is large). Then:*

1. *Firm 1’s optimal investment  $\mu_1^*$  satisfies*

$$C'(\mu_1^*) = Q_1^*,$$

*where  $Q_1^*$  denotes the equilibrium sales of firm 1.*

2. *Firm 2 has zero incentive to innovate.*

This result follows from the fact that when the remain-in constraint is binding, we have  $\frac{dp_1}{d\mu} = 1$ . As a result, an increase in firm 1’s quality translates one-to-one into an *increase* in its equilibrium price, and hence firm 1 invests in direct

proportion to the size of its own market (which, given its winning position, is large). In contrast, an increase in firm 2's quality translates one-to-one into an *reduction* in its rival's price; thus, this higher quality has zero impact on firm 2's revenues.

In practice, firms may also increase the quality of the products by means of acquiring startups with valuable product innovations. Here too, competition for the "in" position may lead to highly asymmetric outcomes. To illustrate, suppose a third party (a "startup") possesses an innovation and, prior to engaging in price competition, firms 1 and 2 bid in a (second-price) auction to buy the startup. Suppose the firm that acquires the startup adopts its innovation, and as a result improves its quality by  $\Delta\mu$ .

In this setting, provided the hypothesis of Corollary 4 is met, firm 1's maximum bid for the startup is  $2\Delta\mu Q_1$ ; whereas firm 2's maximum bid is 0.<sup>7</sup> Therefore, firm 1 acquires the startup and pays 0 for it, further cementing its dominant position. In fact, in a multi-period version of this merger game in which a new startup emerges in every period, the dominant firm outbids its rival for each new startup; thus, its dominant position becomes more and more entrenched as time goes by.

## 4 Piecewise Linear Demand

When consumers' tastes follow a particular type of distribution (see Figure 8a), the in/out demand curve is piecewise linear. Figure 8b depicts the in/out demand curve for a monopolist firm corresponding to Figure 8a.

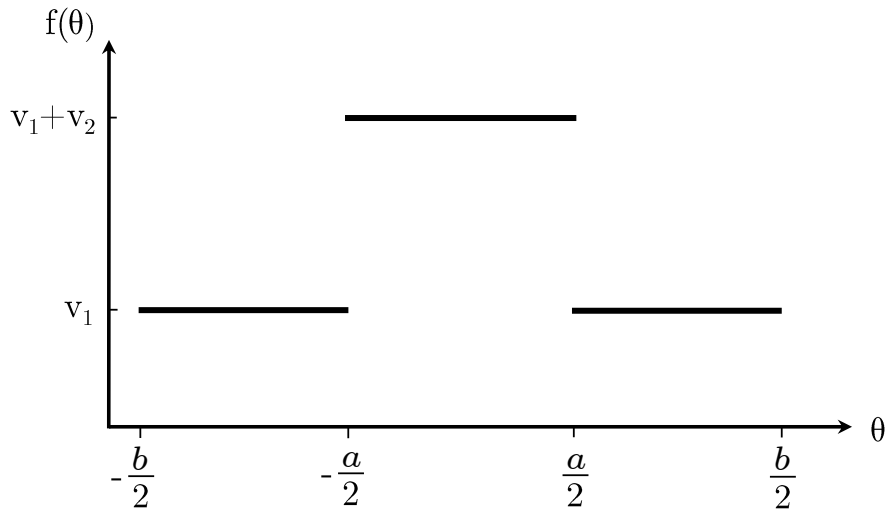
When demand is piecewise linear, we can solve explicitly for the outcome of price competition. Furthermore, piecewise linear demand facilitates an analysis of the effects of demand volatility.

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<sup>7</sup>By winning, firm 1 not only increases its quality by  $\Delta\mu$ , it also prevents firm 2 from increasing its quality by  $\Delta\mu$ . Hence firm 1 is willing to bid  $2\Delta\mu$  per unit of expected sales (as opposed to  $\Delta\mu$ ).



(a) Pdf that gives rise to piecewise linear demand.



(b) Corresponding demand curve for the monopoly case (demand is in/out if  $\alpha > \frac{1}{v_1+v_2}$ ).

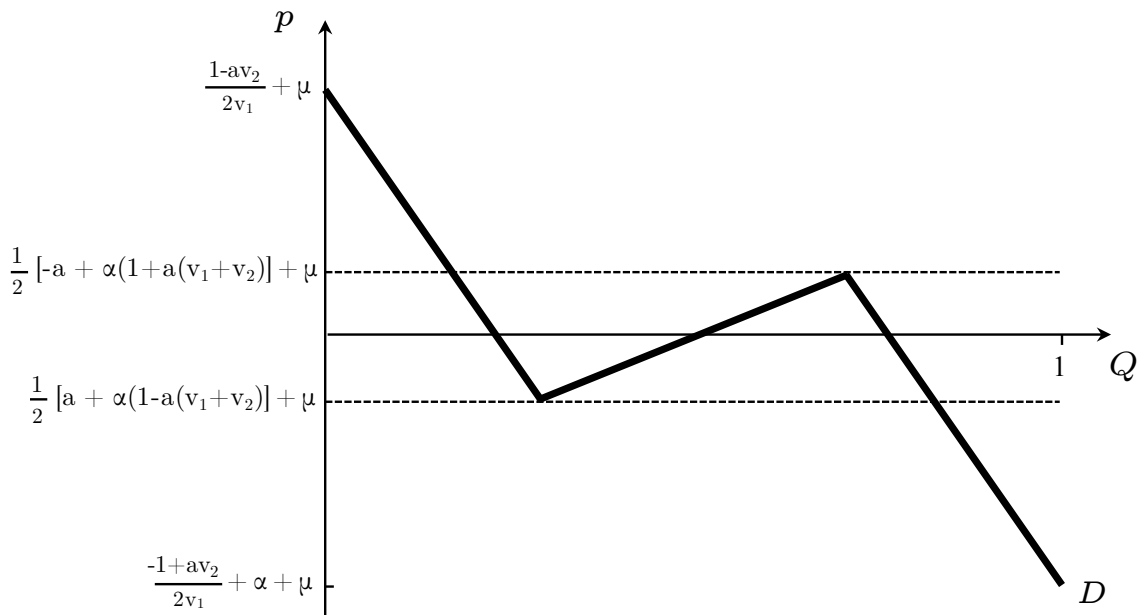


Figure 8

## 4.1 Demand Volatility

Suppose, as in Section 2, there is a monopolist and suppose there is just a single pricing period ( $T = 1$ ). Consumers' tastes are distributed as in Figure 8a and the network externalities are sufficiently large that demand is in/out ( $\alpha > \frac{1}{v_1+v_2}$ ). In contrast to Section 2,  $\mu$  (the quality of the monopolist's good relative to the outside option) is a random variable:  $\mu = \hat{\mu} + \varepsilon$ , where:

$$\varepsilon = \begin{cases} \sigma, & \text{with probability } r. \\ -\sigma, & \text{with probability } r. \\ 0 & \text{with probability } 1 - 2r. \end{cases}$$

The resulting demand curve,  $p^d(Q)$ , has a random component:

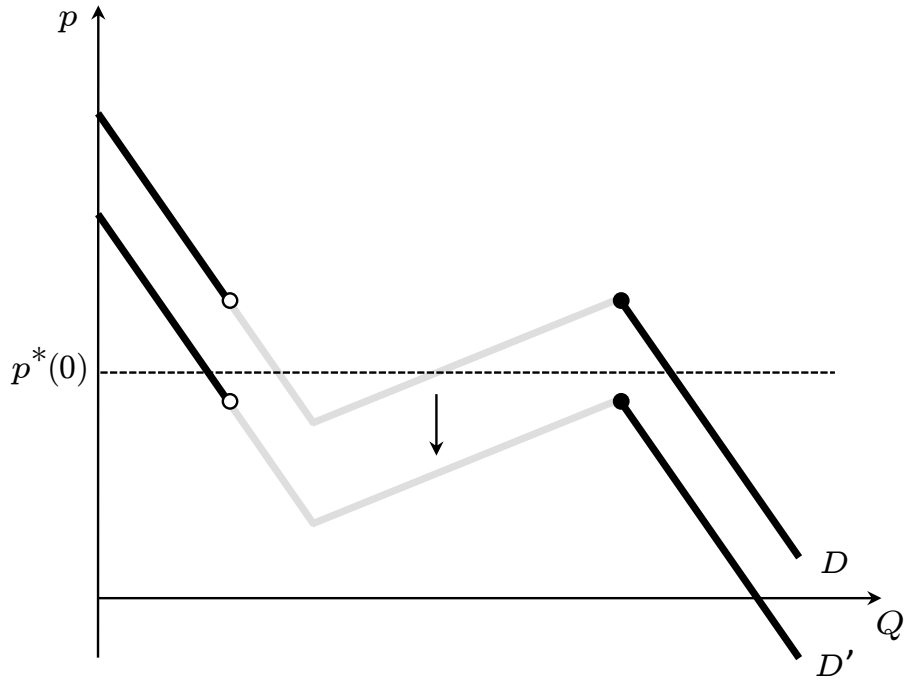
$$p^d(Q) = \hat{p}(Q) + \varepsilon.$$

We assume the monopolist is risk neutral.

How does demand volatility affect optimal pricing? Let  $p^*(\sigma)$  denote the optimal price for a given level of volatility,  $\sigma$ . Let us focus attention on the case where the monopolist is "in" for sure when there is no demand volatility ( $\sigma = 0$ ). Figure 9a illustrates that, if demand is sufficiently volatile, the firm risks going "out" if it keeps its price at  $p^*(0)$ . Going "out" is quite costly as it involves a discontinuous decline in demand. The firm has an incentive to shade the price when demand is volatile to reduce or eliminate the risk of going out.

Figure 9b shows the optimal price as a function of the volatility. When volatility is low ( $\sigma \leq p_{\max} - p^*(0)$ ), the monopolist does not risk going out if it sets a price  $p^*(0)$ . Consequently, the optimal price is simply  $p^*(0)$ . When volatility is in an intermediate range ( $p_{\max} - p^*(0) < \sigma < \bar{\sigma}$ ), the firm shades its price to eliminate the risk of going "out." In this region  $p^*(\sigma) = p^*(0) - [\sigma - (p_{\max} - p^*(0))]$ . When volatility is sufficiently high ( $\sigma > \bar{\sigma}$ ), the cost of shading is sufficiently high that the firm chooses to accept some risk of going "out." In this region, the firm charges

(a) There is a risk the firm goes from “in” to “out” if  $\sigma > 0$ .



(b) Optimal pricing as a function of volatility ( $\sigma$ ).

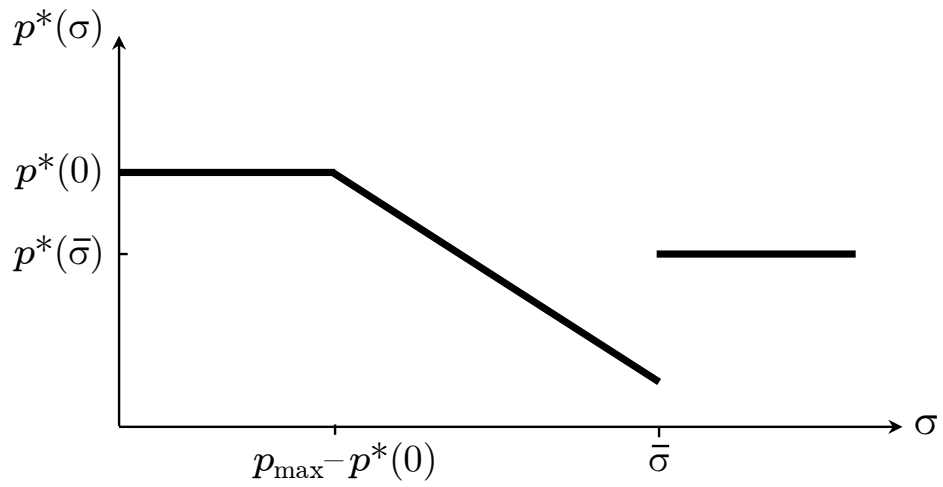


Figure 9

a price  $p^*(\bar{\sigma})$  and it goes “out” with probability  $r$ . The reason  $p^*(\bar{\sigma}) < p^*(0)$  is as follows. While the demand curve has the same slope when the firm is “out” as it does when the firm is “in,” demand is actually more elastic in the “out” region. Hence, the price markup is smaller when there is a chance of going “out.”

## 4.2 Competition

Suppose firms 1 and 2 are engaged in the price competition game described in Section 3. Consumers’ tastes are distributed as in Figure 8a and  $\alpha$  is between  $\alpha_{\min}$  and  $\alpha_{\max}$ , where  $\alpha_{\min} = \frac{1}{2(v_1+v_2)}$  and  $\alpha_{\max} = \frac{1+a(v_1-v_2)}{2v_1(1+a(v_1+v_2))}$ . The condition  $\alpha > \alpha_{\min}$  ensures demand is in/out; the condition  $\alpha < \alpha_{\max}$  ensures  $\Delta^d(Q_1)$  is maximized at  $Q_1 = 0$  rather than on the upward-sloping part of the demand curve.

Under these assumptions, it is possible to solve explicitly for the equilibrium of the pricing game. As Figure 10 shows, the equilibrium of the game depends upon which region parameters  $\mu$  and  $\alpha$  fall into. Proposition 5, stated in the Appendix, specifies the equilibrium prices and quantities in each region.

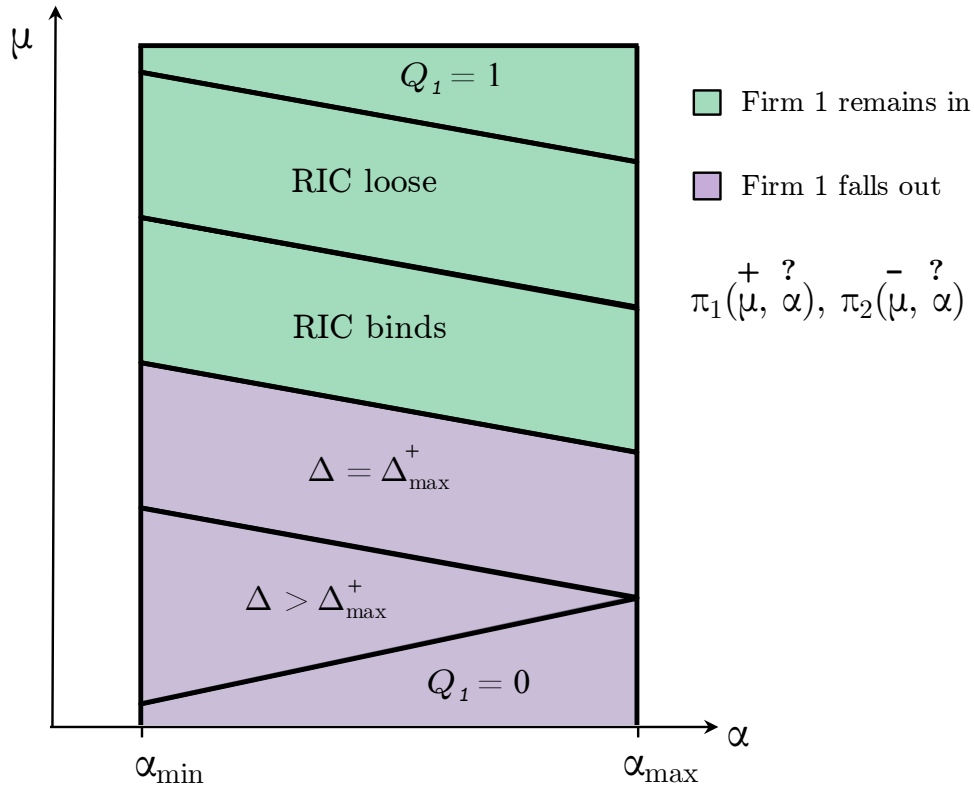
Several points are worth making. First, as one would expect, firm 1 “remains in” if  $\mu$  (good 1’s relative quality) is above a threshold; firm 1 “falls out” if  $\mu$  is below the threshold. The remain-in constraint (RIC) is binding when  $\mu$  is just above the threshold. When  $\mu$  is just below the threshold, firm 2 chooses the minimal price that puts it “in” ( $\Delta = \Delta_{\max}^+$ ).

Second, as one crosses the threshold from the region where RIC is satisfied to the region where RIC is violated, prices jump discontinuously. Firm 1’s price jumps up and firm 2’s price jumps down. Intuitively, prices jump because firm 1 gives up on remaining “in” and firm 2 decides it is worthwhile to go “in.”

Third, competition between the firms is “normal” in the region where RIC is loose and the region where  $\Delta > \Delta_{\max}^+$ . Competition is normal in the following sense: changes in relative quality,  $\mu$ , impact the prices and quantities of both goods. Competition is abnormal when RIC binds or  $\Delta = \Delta_{\max}^+$ . In those regions, changes in  $\mu$  have no effect on quantities and only affect firm 1’s price.

Finally, one might think that the firm that ends up “in” benefits from an in-

## Equilibrium Outcome: Piecewise Linear Demand



**Figure 10**

crease in network externalities ( $\alpha$ ). In fact, it is ambiguous whether an increase in  $\alpha$  benefits or hurts the “in” firm. The reason is as follows. An increase in  $\alpha$  has two effects (which are illustrated in Figure 11):

1. For a given price differential, the “in” firm gets a larger share of the market.
2. Demand is more elastic.

The “in” firm benefits from the first effect. The second effect, however, can drive more intense competition between the firms for the “in” position. This competition may be harmful to the “in” firm. A lesson is that, even in cases where one firm has a dominant market share, it may be incorrect to assume that competition

is weak.

How an increase in  $\alpha$  affects demand.

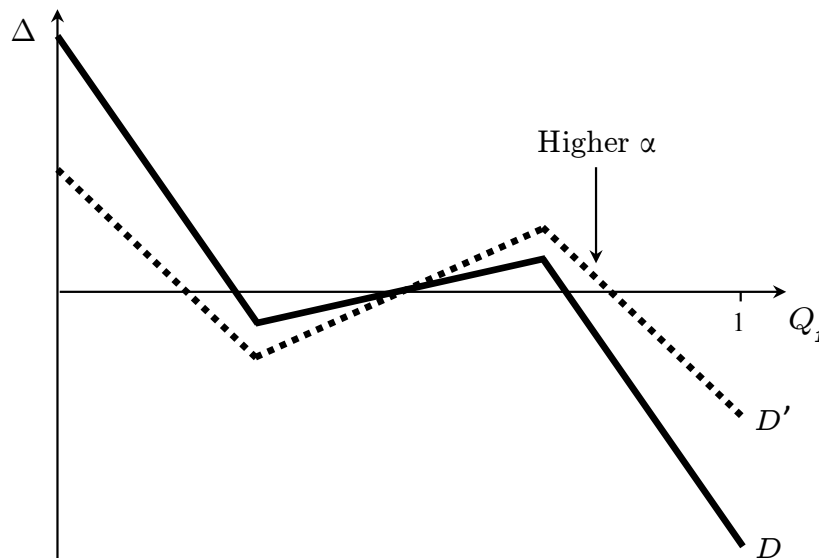


Figure 11

## 5 Conclusion

We proposed a rich yet tractable model to study optimal pricing and price competition in the presence of network effects. Our model, which builds upon Katz and Shapiro's seminal work, extends and micro-founds Becker's theory of restaurant pricing in which restaurants face a non-standard demand curve with alternating regions of negative and positive slope. As we show, such demand curves arise naturally when the population distribution of consumers is single-peaked and network externalities are strong.

A critical feature of markets with networks externalities, embedded in the above non-standard demand, is their ability to generate multiple equilibria. Such multiplicity is behind large asymmetries between winners and losers. It is also behind the apparent paradox that it is quite difficult to become a winner, and yet the winning position is fragile; hence, winners are not asleep.

Understanding markets with network externalities therefore requires understanding how consumers pick one equilibrium over another. To this end, we proposed a simple theory of equilibrium selection (formally, *introspective equilibrium*) that captures the notion that a firm's popularity exhibits a form of inertia over time, and is affected as well by salient consumers that are popular among their peers. A firm's default popularity, inherited from the previous period, then determines whether it currently faces its worst possible demand curve (the "out" demand), its best possible curve (the "in" demand), or an intermediate version of the two (the "between" demand). Each of these demand curves has a well-behaved shape with a standard negative slope but with a discontinuity. This simple classification immediately sheds light on the firm's optimal pricing, its equilibrium transitions between the losing and the winning positions, and its incentives for innovation.

Our model features a form of asymmetric competition in which a winning and losing firm co-exist, with the losing firm keeping the winning firm in check. We show that this check on the winning firm is a generalized form of "limit pricing," whereby a monopolist is disciplined by a potential entrant.<sup>8</sup> In our case, rather than deterring entry outright, the winning firm needs to deter the losing firm from becoming popular. It does so by allowing the losing firm to enjoy rents from a small but loyal consumer base, a form of consolation prize.

In subsequent work, we expect to propose a method for valuing firms in the presence of network effects, with such effects opening the possibility of large profits for popular firms, but also to inevitable risks of sudden failure.

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<sup>8</sup>For classic references on limit pricing, see Gaskins (1971) and Milgrom and Roberts (1982).

## 6 Appendix

**Proposition 5.** *The equilibrium of the pricing game depends upon which region parameters  $\mu$  and  $\alpha$  fall into. There are six regions which can be ordered from a highest- $\mu$  region to a lowest- $\mu$  region:*

1. *RIC is loose and  $Q_1^* = 1$  ( $\mu \geq \bar{\mu}_1$ ):*

$$p_1^* = \mu + \alpha + \frac{-1 + av_2}{2v_1}.$$

$$p_2^* = 0.$$

2. *RIC is loose and  $Q_1^* < 1$  ( $\bar{\mu}_1 > \mu \geq \bar{\mu}_2$ ):*

$$p_1^* = \frac{1}{2}\mu - \frac{3}{2}\alpha + \frac{3 + av_2}{4v_1}.$$

$$p_2^* = -\frac{1}{4}\mu - \frac{5}{4}\alpha + \frac{5 - av_2}{8v_1}.$$

$$Q_1^* = \frac{v_1}{2(1 - 2v_1\alpha)} \left( \frac{1}{2}\mu - \frac{3}{2}\alpha + \frac{3 + av_2}{4v_1} \right).$$

3. *RIC binds ( $\bar{\mu}_2 > \mu \geq \bar{\mu}_3$ ):*

$$p_1^* = \mu + \alpha(2a(v_1 + v_2) - 1) + \frac{a(3v_2 - 2v_1) + 1}{2v_1}$$

$$- \frac{2}{v_1} \sqrt{av_2 \left[ \frac{1}{2}(1 + a(v_2 - v_1)) - \alpha v_1(1 - a(v_1 + v_2)) \right]}.$$

$$p_2^* = \alpha(a(v_1 + v_2) - 1) + \frac{a(v_2 - 2v_1) + 3}{4v_1}$$

$$- \frac{1}{v_1} \sqrt{av_2 \left[ \frac{1}{2}(1 + a(v_2 - v_1)) - \alpha v_1(1 - a(v_1 + v_2)) \right]}.$$

$$Q_1^* = \frac{1 - a(v_2 - v_1)}{2(1 - 2v_1\alpha)} - \frac{\alpha v_1(a(v_1 + v_2) + 1)}{1 - 2v_1\alpha}$$

$$+ \frac{1}{1 - 2v_1\alpha} \sqrt{av_2 \left[ \frac{1}{2}(1 + a(v_2 - v_1)) - \alpha v_1(1 - a(v_1 + v_2)) \right]}.$$



4.  $\Delta = \Delta_{\max}^+$  ( $\bar{\mu}_3 > \mu \geq \bar{\mu}_4$ ):

$$p_1^* = \mu + \alpha(2a(v_1 + v_2) - 1) + \frac{1 + a(v_2 - 2v_1)}{2v_1}.$$

$$p_2^* = \alpha(a(v_1 + v_2) - 1) + \frac{1 + a(v_2 - v_1)}{2v_1}.$$

$$Q_1^* = \frac{1 - a(v_2 - v_1) - 2v_1\alpha(1 + a(v_1 + v_2))}{2(1 - 2v_1\alpha)}.$$

5.  $\Delta > \Delta_{\max}^+$  and  $Q_1^* > 0$  ( $\bar{\mu}_4 > \mu \geq \bar{\mu}_5$ ):

$$p_1^* = \frac{1}{2}\mu - \frac{3}{2}\alpha + \frac{3 - av_2}{4v_1}.$$

$$p_2^* = -\frac{1}{4}\mu - \frac{5}{4}\alpha + \frac{5 + av_2}{8v_1}.$$

$$Q_1^* = \frac{v_1}{2(1 - 2v_1\alpha)} \left( \frac{1}{2}\mu - \frac{3}{2}\alpha + \frac{3 - av_2}{4v_1} \right).$$

6.  $\Delta > \Delta_{\max}^+$  and  $Q_1^* = 0$  ( $\mu < \bar{\mu}_5$ ):

$$p_1^* = 0.$$

$$p_2^* = -\mu + \alpha - \frac{1 - av_2}{2v_1}.$$

*The cutoffs between regions are defined as follows:*

$$\bar{\mu}_1 = \frac{3 - av_2}{2v_1} - 3\alpha.$$

$$\bar{\mu}_2 = \frac{-1 + a(3v_2 - 4v_1)}{2v_1} + \alpha(4a(v_1 + v_2) + 1).$$

$$\begin{aligned} \bar{\mu}_3 &= \frac{a(5v_2 - 4v_1) - 1}{2v_1} + \alpha(1 + 4a(v_1 + v_2)) \\ &\quad + \frac{-1 - 3a(v_2 - v_1) + 2\alpha v_1(1 - 3a(v_1 + v_2))}{v_1[1 + a(v_2 - v_1) + 2\alpha v_1(-1 + a(v_1 + v_2))]} \end{aligned}$$

$$\begin{aligned} \bar{\mu}_4 &= \frac{a(5v_2 - 4v_1) - 2}{2v_1} + \alpha(4a(v_1 + v_2) + 1) \\ &\quad - \frac{4}{v_1} \sqrt{av_2 \left[ \frac{1}{2}(1 + a(v_2 - v_1)) - \alpha v_1(1 - a(v_1 + v_2)) \right]}. \end{aligned}$$

$$\bar{\mu}_5 = \frac{-5 + av_2}{2v_1} + 5\alpha.$$

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